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ABSTRACT

This monograph is written for teachers of students in grades seven to nine to heighten awareness of the skills in the measurement and geometry cluster of the New Jersey High School Proficiency test (HSPT) and to suggest strategies for teaching them. Eleven skills in the HSPT measurement and geometry cluster are grouped according to type of cognitive task. The tasks are presented in order of increasing complexity. The four types of tasks are: identifying, computing, applying a formula, and simplifying. A framework for instruction is included. The statement of the skill, the statement of the conditions for testing, actual test items, enabling skills, and instructional activities are included for each of the skills. A list of geometry resources and a table of state averages of the 11 skills of the HSPT in 1984 and 1985 are appended. (YP)

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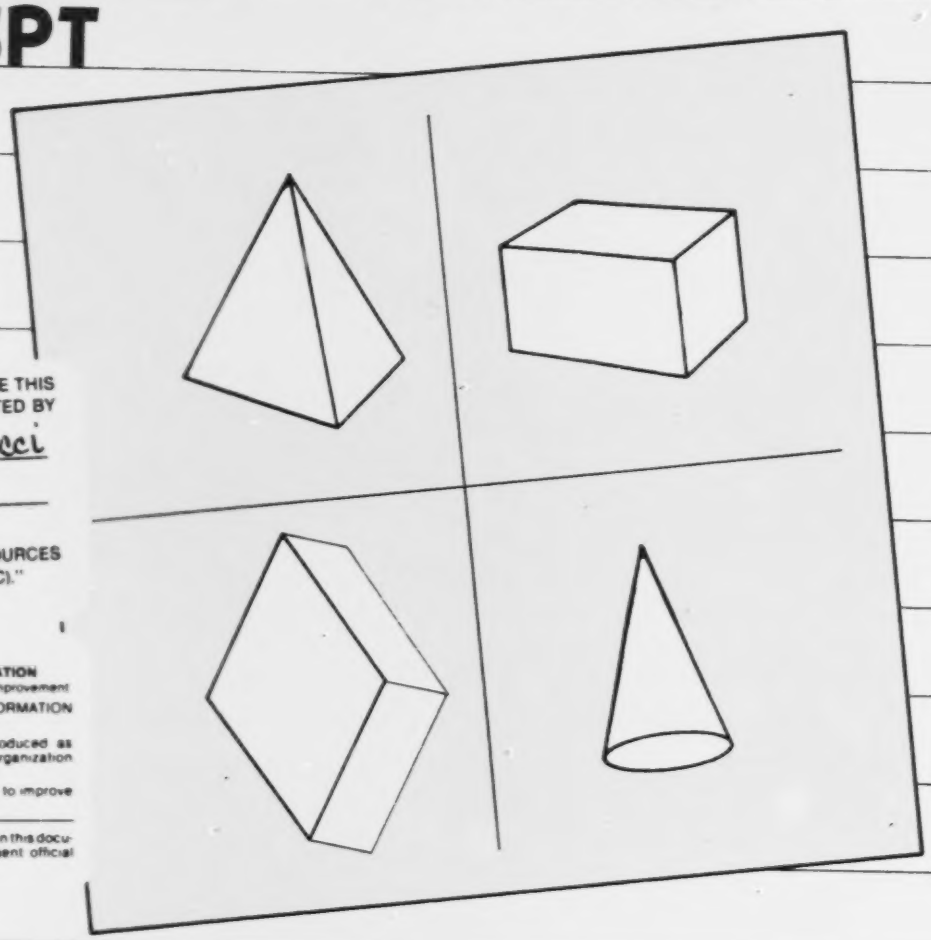
Improving Mathematical Skills Assessed on the HSPT



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Measurement and Geometry

TEACHING LEARNING TESTING

New Jersey Department of Education, Division of General Academic Education

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IMPROVING MATHEMATICAL SKILLS ASSESSED ON THE HIGH SCHOOL PROFICIENCY TEST

MEASUREMENT AND GEOMETRY

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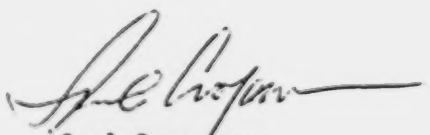
COMMISSIONER'S FOREWORD

The High School Proficiency Test (HSPT) is a primary tool being used in New Jersey to raise academic standards for public school students. Student achievement on the mathematics portion of the HSPT has caused grave concern to educators, parents and the general public since the results of the first administration of the test were made public in 1984.

On the mathematics section of the High School Proficiency Test in 1984, Measurement-Geometry had the lowest mean cluster score of the seven clusters (50 percent). In previous years, the Minimum Basic Skills (MBS) test scores on the measurement-geometry clusters were consistently lower than scores for computation and number concepts. For example, only 70 percent of the ninth grade students correctly identified a right angle.

The HSPT State Report: 1983-84 Analysis and Interpretation of Statewide Performance pointed out the need for "earlier and sequential instruction at all grade levels" and "activities that are practical and use hands-on experience." These recommendations must now be acted upon.

The purpose of this monograph is to provide teachers in grades seven through nine with strategies that are supported by practitioners and researchers and that promote understanding and performance in measurement and geometry skills. This monograph is one of a series being developed by the department to improve mathematical skills assessed on the High School Proficiency Test. The strategies recommends have been successfully field tested through training activities conducted by the department at the Regional Curriculum Services Units.



Saul Cooperman
Commissioner

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INTRODUCTION

This monograph is written for teachers of students in grades seven to nine to heighten awareness of the skills in the measurement and geometry cluster of the New Jersey High School Proficiency Test (HSPT) and to suggest strategies for teaching them.

Reports from the National Assessment of Educational Progress and the Second International Mathematics Study recommend that mathematics instruction for seventh and eighth grade students involve learners with concrete materials, focus on concept development, and be targeted to desired outcomes. In New Jersey, the desired outcome is student achievement of the mathematics skills tested on the High School Proficiency Test. The strategies recommended in this monograph focus on concept development and on activities that use many types of concrete materials.

The content of this monograph is based on the HSPT Skill Array Mathematics. The sample test items are drawn from the HSPT Skill Array and the test specifications. The skills are restated in this monograph as a reference to help teachers plan to include these skills in their mathematics courses.

Board policies, district adopted curricula, and textbooks determine the broad framework of instruction in mathematics, but teachers decide when to teach measurement and geometry, how much time they spend on the topic, and which skills to emphasize. The following suggestions from recent research and national reports can help teachers make these important decisions. Teachers can become more effective if they:

- Become familiar with the skills to be tested at ninth grade and the benchmark skills identified in the HSPT Skill Array booklet.
- Decide the skills upon which to focus instruction in the time available. (All H.S.P.T. skills need to be taught and reviewed within the three year cycle from seventh to ninth grade).
- Check the match between the skills presented in the texbook, their curriculum guide and the HSPT Measurement/Geometry Cluster.
- Plan to teach units on measurement and informal geometry early in the year.
- Plan all instruction in measurement and geometry to move from concrete experiences to the abstract, and to involve students in visualizing, verbalizing, constructing, and deductive reasoning.
- Assemble tools of measurement including scales, rulers, objects to be measured, weights, and calibrated containers for both metric and standard units.

- o Assemble models of cones, cylinders, spheres, rectangular solids and two-dimensional shapes.

- o Provide for follow-up to include applications and problem solving using these measurement and geometry skills.

OVERVIEW OF THE MONOGRAPH

The eleven skills in the HSPT measurement and geometry cluster are not presented in an instructional sequence in the Skills Array. In this monograph the skills are grouped according to a type of cognitive task. The tasks are presented in order of increasing complexity. Instruction could be organized by type of cognitive task. The four types of tasks are: identifying, computing, applying a formula, and simplifying. The HSPT skills are numbered as they appear in the Skill Array. They are listed below with that identifying number under each type of task.

IDENTIFYING TASKS: The following skills require a specific word (label) to be applied to a visual or verbal description (definition).

Skill #22	Identify and define acute, obtuse and right angles
Skill #23	Demonstrate an understanding of complementary and supplementary angles
Skill #29	Identify similar figures
Skill #30	Identify points on a coordinate plane
Skill #31	Demonstrate an understanding of parallel and perpendicular lines
Note:	Skill #23 and skill #31 are tested as identifying tasks. Skill #23 also requires computing.

COMPUTING TASKS: The following skills require the selection of operations and application of the knowledge of vocabulary, of relationships among the units of measure, and of characteristics of the geometric figures.

Skill #21	Add, subtract, multiply and divide measures using metric and U.S. standard units
Skill #23	Demonstrate an understanding of complementary and supplementary angles
Skill #24	Demonstrate an understanding that the sum of the measures of the angles of a triangle is 180°
Skill #26	Find the perimeter of a polygon

Skill #28 Compute the area of an irregular figure

Note: Skill #28 requires both computing and simplifying tasks.

APPLYING A FORMULA TASKS: The following two skills require comprehension of symbols, letters, and numbers as well as the meaning attached to their juxtaposition in the formula. Instructional strategies that improve understanding of the symbols of the formula should be stressed rather than rote memorization. Then fewer errors will occur when applying a formula.

Skill #27 Compute the area of a specified region

Skill #25 Find the volume of a cone, pyramid, rectangular solid or sphere

SIMPLIFYING COMPLEX TASKS: The strategy suggested for these skills is breaking the complex figure into simple figures. In the case of volume, the relation of volume to area may be understood when the figure is "dissected" to show its surface area.

Skill #28 Compute the area of an irregular figure

Skill #27 Compute the area of a specified region (given two figures, one embedded)

Skill #25 Find the volume of a cone, pyramid, rectangular solid or sphere

Several of the skills in the HSPT measurement and geometry cluster require more than one of the tasks described above. These "higher order" skills require the student to use more than one cognitive task to be able to demonstrate mastery of them. To assist students with these types of problems, instruction must focus on helping them organize the information given and select the appropriate knowledge and processes for solution.

By the time these skills are tested in the ninth grade, students must have been taught and have mastered vocabulary and symbols as well as the ability to compute ability. To help teachers prepare students to achieve these skills, some enabling skills are listed for each HSPT skill. Teachers need to assess students' prior knowledge and skills as they plan instruction. For some students, instruction may need to begin with these enabling skills.

In this monograph, several activities are presented for each HSPT skill. The activities represent selections from teachers, authors, workshop leaders, documents and professional journals. Each activity was selected because it fits in the developmental framework of measurement or geometry learning theory or because it represents an example of the use of materials suggested by the research. Textbooks and commercially produced supplemental materials commonly in use provide many paper and pencil

activities. The activities presented here focus on materials that help develop concepts, vocabulary, and a transition from concrete to abstract experiences, such as graph paper, geoboards and base ten blocks.

For each skill in the measurement and geometry cluster this monograph includes:

- o The statement of the skill
- o The statement of the conditions for testing
- o Actual test items
- o Enabling skills
- o Instructional activities

A FRAMEWORK FOR PRESENTING MEASUREMENT AND GEOMETRY INSTRUCTION

The skills which are identified in the HSPT measurement and geometry cluster do not require formal proofs, and are therefore representative of informal geometry and of measurement skills applied to real experiences. With the possible exception of finding the volume of a set of solids, all of these skills are introduced in the curriculum by eighth grade. Yet, according to HSPT test results, many students do not master these skills by ninth grade. Cognitive style differences and development among students in seventh and eighth grade may partially account for the lack of achievement, but the presentation of mathematical ideas as a bridge to and from the student's real world is critical to success in learning informal geometry and this is a factor which teachers can control.

One framework for presenting instruction is suggested by recent studies of a hypothetical model of mental development in geometry named after two Dutch educators, the van Hieles. The van Hieles propose that a combination of time, content, and teaching methods can carry students through these levels of geometric understanding. This is a brief description of that model.

Level 1. Recognition-Visualization. The student names a figure and can react to the total figure and copy it.

Level 2. Analysis. The student analyzes properties of a figure. Through observations, measurements, drawings and acting on models, the properties of a particular figure are realized.

Level 3. Ordering-Abstraction. The student is able to order figures based on properties of concepts, and begins to understand interrelationships and the role of definitions. A square can be viewed as having four interior right angles. Deductive methods appear in conjunction with experimentation.

Level 4. Deduction. The student reasons formally within a mathematical system. The role of postulates, theorems, proofs and undefined terms is understood.

Level 5. Rigor. The student understands the importance of precision in dealing with axioms and postulates. The student can compare systems and study geometries in the absence of concrete examples. This level of sophisticated thinking is rarely reached by high school students. (Burger, 1985)

The activities in this monograph have been selected to promote teaching methods that will carry students through these levels of development. This collection represents only a sample of possible activities. Teachers are urged to search for others that meet the needs of their students. The strategies suggested in this monograph focus on problem solving as a process that encourages inference and deduction. Vocabulary, formulas, and rules of arithmetic operations need to be memorized, but the focus should be on their role as tools for problem solving, and not as ends in themselves.

HSPT Mathematics Skills and Supporting Activities

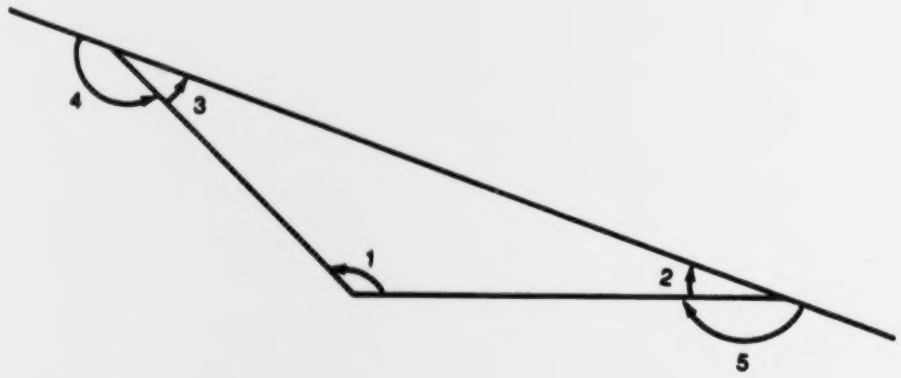
SECTION ONE
IDENTIFYING AND DEFINING TASKS

The following skills, #22, #23, #29, #30 and #31, are grouped together in section one because they are identifying tasks. Students who demonstrate behavior in van Hiele Level 1 can perform these tasks with instruction and practice.

Skill #22

Identify and define acute, obtuse and right angles. The conditions for testing include both visual and verbal formats.

Test item:



In the figure above, which two angles are acute?

(A.) 2 and 3
B. 1 and 5
C. 3 and 4
D. 2 and 5

An angle that measures 90° is

A. an acute angle.
B. an obtuse angle.
(C.) a right angle.
D. a supplementary angle.

(HSPT, 1985)

Enabling Skills and Knowledge

Give students the following information as needed:

1. Rays, segments, or lines intersect to form angles.
2. Angles may be identified and labeled with numbers or letters.
3. Angles may also be identified by three letters with the vertex in the middle. Example: $\angle$ ABC
4. An angle is measured in units called degrees.
5. The symbol for degree is °.
6. The symbol for angle is $\angle$.
7. A protractor is used to measure angles. (Students are able to measure angles with a protractor.)
8. The complete rotation around the center of a circle measures 360 degrees (360°)
9. Perpendicular lines form 90° angles.
10. A 90° angle is called a right angle.
11. An angle which measures less than 90° is called an acute angle.
12. An angle that measures more than 90° and less than 180° is called an obtuse angle.

Instructional Implications

Instruction in informal geometry must include objectives to teach the language of mathematics. Reading, writing, and orally communicating mathematics are important skills. One of those skills is the ability to translate from numbers to words, and from words to numbers or symbols. Activities that direct students to describe figures they see and to draw figures from verbal descriptions will enhance their knowledge of geometry. The sample of activities that follow are keyed to van Hiele Levels 1 and 2, as described in the Introduction.

Angle Activity 1. EXPERIENCING ANGLES AND THEIR PROPERITIES

PURPOSE: To sharpen students' ability to visualize and analyze properties of a figure (van Heile Levels 1 and 2). To identify and draw students' atten- tion to the relevant and most frequently occurring irrele- vant characteristics of angles by presenting both examples and non-examples.

MATERIALS: A variety of examples and non-examples of types of angles

DIRECTIONS:

A. Teacher: Show examples and non-examples of each type of angle.

(a) (b) (c) (d) (e)

Examples **Non-examples**

Teacher: Rotate figures to represent angles in many orientations. Use cutouts of angles to easily change orientation on the overhead or chalkboard. Identify figures by a reference letter or number.

Present a written list of words describing types of angles and non-examples.

right angle

obtuse angle

acute angle

straight angle (Note: Included for comparison, although not on the HSPT skill statement.

Students: Match the figures shown to the correct words. Example: (a) above, right angle

B. Teacher: Alter the procedure above by giving words (printed and/or oral) that describe each angle.

Students: Match the words to diagrams of angles (van Heile Level 1), or draw (van Heile Level 2) angles to represent words.

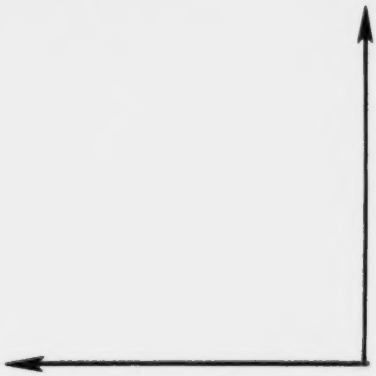
C. Teacher: Have students make flash cards using 3x5 index cards for memory/review, such as the ones on the following page.

STRAIGHT ANGLE

A straight angle has 180 degrees.

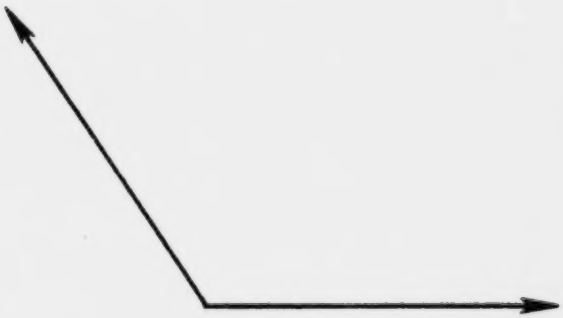
RIGHT ANGLE

A right angle measures 90 degrees. The two rays are perpendicular.



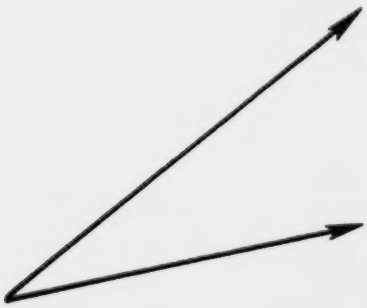
OBTUSE ANGLE

An obtuse angle is one that contains more than 90 degrees but less than 180 degrees.



ACUTE ANGLE

An acute angle is less than a right angle (less than 90 degrees).



TEACHER: Have students use flash cards as a sorting game. Have students cut the cards in half to mix and match the printed definitions with the angles shown.

The next two learning experiences promote the development of analysis, as described by van Hiele as Level 2.

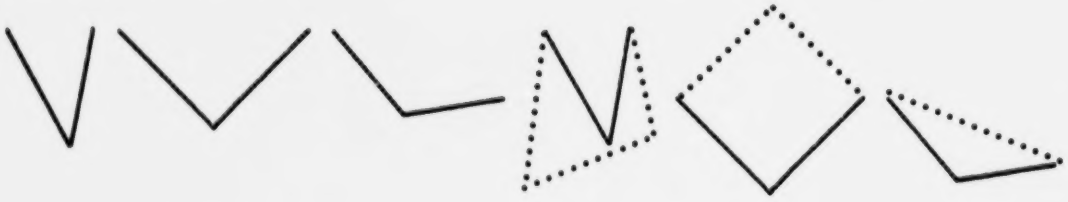
D. Teacher: Present figures with interior angles: triangles, hexagons, rectangles. Locate and label interior angles of each figure using arcs.

Students: Expand labeling activity by observing the angles as parts of figures. Examine each figure and count the number of angles and the types of angles which make up a given figure. Estimate and then compare the measure of the angles of each figure. Draw figures, then cut them apart and reassemble them as puzzle pieces. Exchange puzzle pieces with a partner. (optional)

E. Teacher: Give several examples of each type of angle. Discuss open figures and closed figures.

Students: Complete figures by closing with any number of straight line segments. Identify the number of each kind of angle represented in each closed figure.

<u>Samples of Angles Given</u> <u>Students' Possible Solutions</u>



Angle Activity 2. MAKING ANGLES FROM STICKS

PURPOSE: To experience the relationship of angles with a common vertex by manipulation. To stimulate the need to identify and label.

MATERIALS: stirer sticks or firm paper strips, thumb tacks or brads

DIRECTIONS: Teacher can demonstrate and/or present this activity orally.

Direct students to take two strips or stirer sticks and connect them with a thumb tack or brad so that the strips are joined in the middle. Show students how to form right angles using the sticks. Ask, "How many right angles are formed?" Show students how to form acute angles. Ask, "How many are formed?" Have students describe how they look. Accept descriptions. Encourage further description as needed. Ask, "Are there other angles already formed?" Angles are either acute, right, obtuse, or straight. Ask, "How many obtuse angles can be formed?" Direct students to trace the arrangements on paper.

Give instructions to label the angles using letters, or numbers as in the HSPT test item.

Name the angles using this vocabulary:

acute

obtuse

straight

right

How many acute angles are shown?

How many obtuse angles are shown?

Observe and listen to students' oral responses to assess students' enabling skills and knowledge. Continue directed activity until students do not have difficulty manipulating strips to form angles. Then determine:

1. Is "correct labeling" instruction necessary? If yes, use Angle Activity 1C.

2. Is symbol reading and writing instruction necessary? If yes, use Angle Activity 3.

Angle Activity 3. LISTEN AND SKETCH, SEE AND SKETCH

PURPOSE: To practice word, symbol (figure) exchange.

MATERIALS: straight edge, paper, pencil

DIRECTIONS: A. Approach this activity as a dictation exercise.

Teacher to students: "Draw the figures described by these words."

1. A right triangle with equal legs.
2. A triangle containing an obtuse angle.
3. A closed figure with two acute angles.
4. A right triangle with one leg approximately 10 times as long as one other leg.
5. Others: teacher's choice.

B. Change the above approach to See and Sketch

Present descriptive statements on charts, overhead, or worksheet.

1. Have students read descriptions using a combination of words and symbols and translate to sketches.
2. Have students label sketches to match printed statements.
3. Select examples based on individual or group needs. Pairing students to exchange drawings and have the partner write the descriptive statement provides peer assessment, feedback, and cooperative learning.

Source: Adapted from a Workshop Activity given by D. Glatzer, N.C.T.M., April 1984

Angle Activity 4. WHICH SHAPE IS THE BEST BUY?

PURPOSE: To develop awareness of figures as bearers of specific properties (angles).

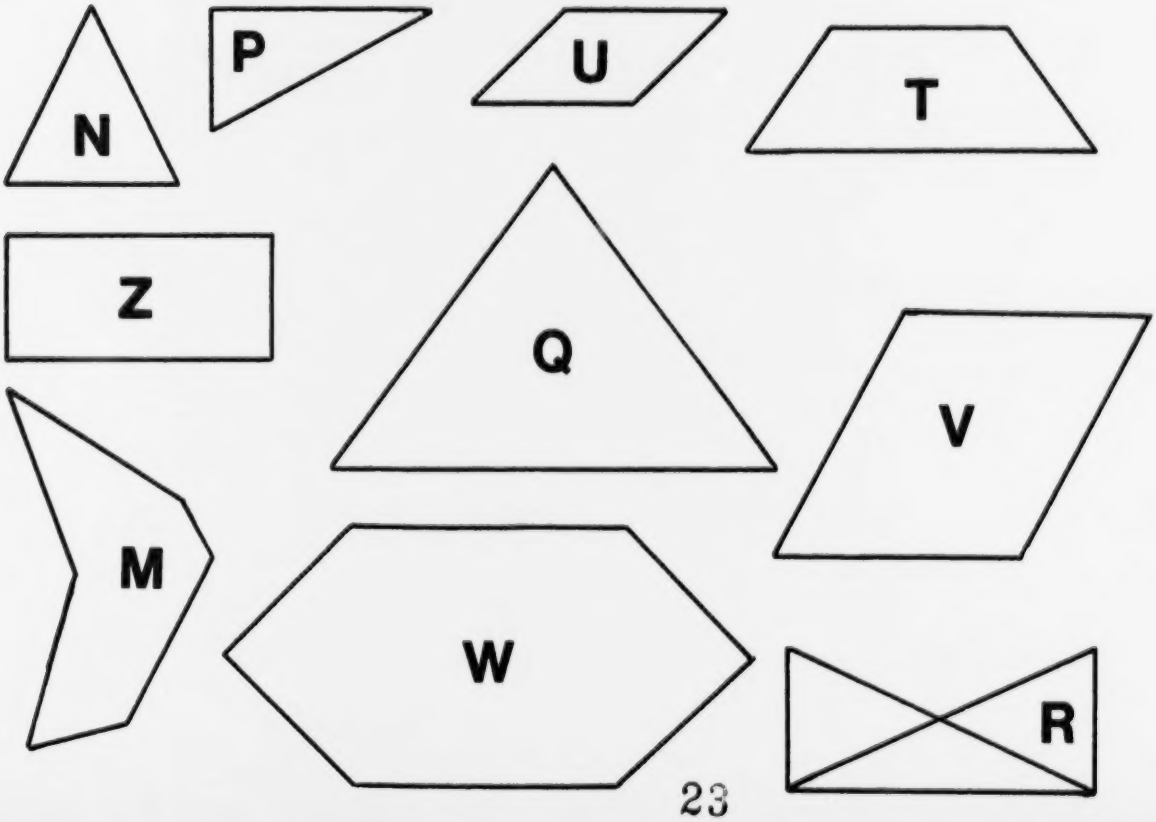
MATERIALS: Sample of regular and irregular shapes on transparency or worksheet, "story" questions, protractor.

DIRECTIONS: Tell students this "story":

A stone cutter prices the pieces of stone that he cuts by the sum of the measure (in degrees) of each of the interior angles. "Degreeables" are reported to be the unit of exchange in the stone cutter's society.

Given below is a sample set of shapes. Estimate the price of each shape in "degreeables" based on the sum of the interior angles.

Which shapes cost the least?
Which shapes cost the most?
Is there a pattern to the costs?
If each shape were doubled in size (area) how would the price change?
Decide on an amount of money, in degreeables, you would spend. Draw a shape that would cost that amount based on the stonecutter's rule.

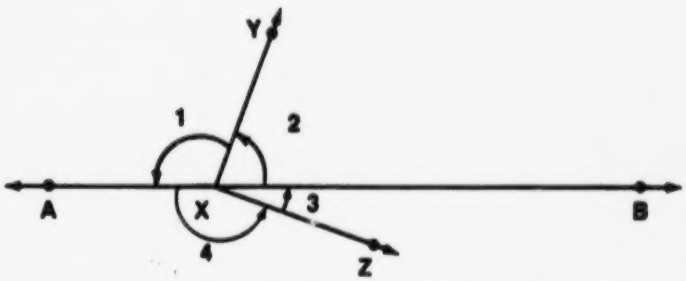


Skill #23

Demonstrate an understanding of complementary and supplementary angles. This skill may be tested by showing a figure with parallel lines cut by a perpendicular line or a transverse line, or by giving a description of an angle which measures less than 90 degrees.

Test Items:

Given: AB is a straight line. $\overline{XY}$ is perpendicular to $\overline{XZ}$.



Which pair of angles is complementary?

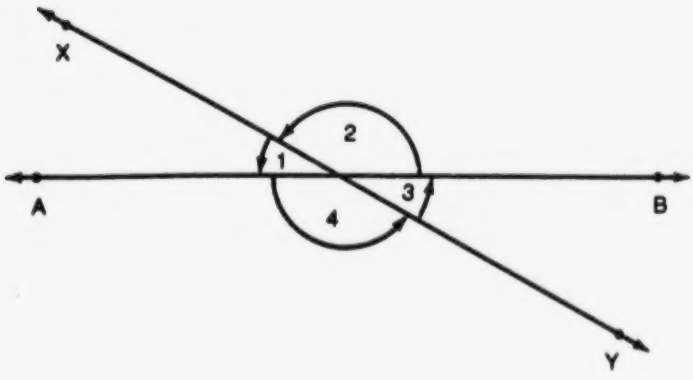
A. ∠1 and ∠3
B. ∠1 and ∠4
(C.) ∠2 and ∠3
D. ∠3 and ∠4

Which pair of angles is supplementary?

(A.) ∠1 and ∠2
B. ∠1 and ∠3
C. ∠2 and ∠3
D. ∠2 and ∠4

HSPT, 1984

Given: AB is a straight line
XY is a straight line



If the sum of two angles is 180° they are

A. complementary angles.
B. obtuse angles.
C. right angles.
(D.) supplementary angles.

If angle 3 measures 35°, what is the measure of angle 4?

A. 55°
B. 135°
(C.) 145°
D. 325°

HSPT, 1985

Enabling Skills and Knowledge

Give students the following information as needed:

1. If the sum of the measures of two angles is 180°, they are supplementary angles.

2. Any angle is the supplement of another angle if the sum of the measures of the angles is 180 degrees.

3. Supplementary angles do not need to share a vertex or a side.

4. If the sum of the measures of two angles is 90°, they are complementary / s.

5. Any angle is the complement of another angle if the sum of the measures of the angles is 90 degrees.

6. Complementary angles do not need to share a vertex or a side.

Instructional Implications

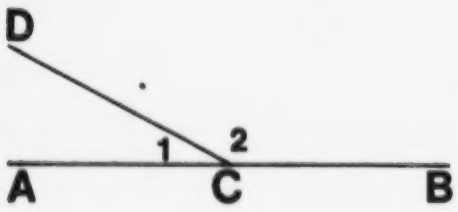
The incorrect responses (distractors) chosen by students in the 1984 administration of the test indicate misuse of the terms complementary and supplementary. About one-third of the students thought that the angle formed by perpendicular lines measured one hundred degrees.

Instruction must focus on correct vocabulary and labeling and provide experience with the different types of angles and their measure.

The following figures are presented as examples of complementary and supplementary angles. Similar worksheets may be given to students to use as a notebook reference. The use of non-examles helps students focus on relevant characteristics.

Supplementary angles

Examples

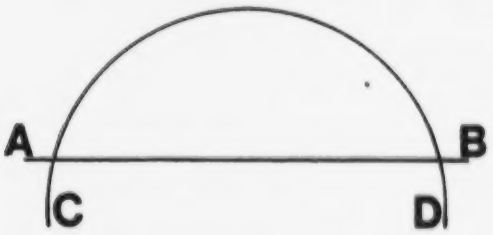


AB is a straight line

CD is a straight line

Therefore ∠1 and ∠2 are supplementary.

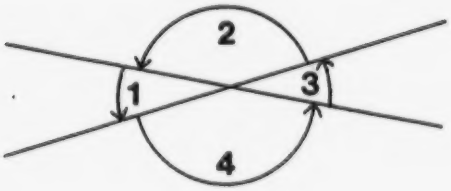
Non-examples



AB is a straight line

CD is not a straight line

These are not examples of supplementary angles.



∠ 1 plus ∠ 2 = 180°

∠ 2 plus ∠ 3 = 180°

∠ 3 plus ∠ 4 = 180°

∠ 4 plus ∠ 1 = 180°

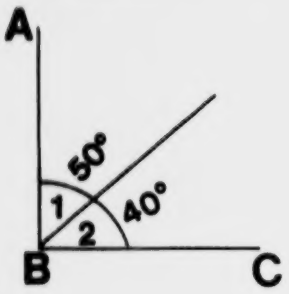
These angles are supplementary

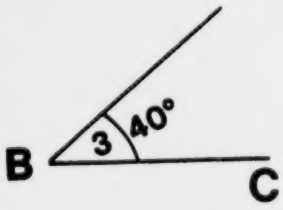
∠ 1 and ∠ 3 are not supplementary

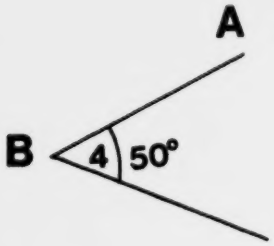
∠ 2 and ∠ 4 are not supplementary.

Complementary Angles

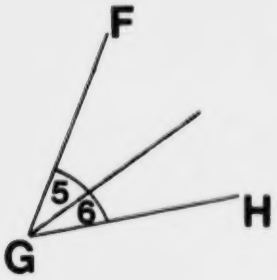
Examples

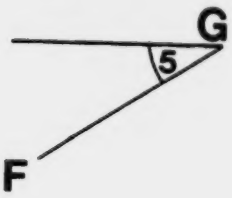


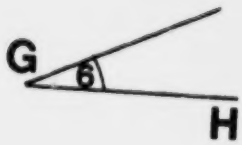




Non-examples







$\angle$ ABC is a right angle

$\angle$ 1 and $\angle$ 2 are complementary

Also, $\angle$3 is 40°, and $\angle$4 is 50°.
They are complementary.

$\angle$ FGH is not a right angle

$\angle$ 5 and $\angle$ 6 are <u>not</u> complementary

Note: Discuss the fact that these labels are used to describe the relationship between <u>two</u> angles only.

Complementary/Supplementary Angle Activity 1. CONSTRUCTING ANGLES

PURPOSE: To increase awareness of interrelationships of figures by experiencing detailed properties through constructions. Ordering and Abstraction, van Heile Level 3, is acquired through this type of experimentation.

MATERIALS: Protractor.

DIRECTIONS: Have students use a protractor to construct a set of examples and non-examples such that the following conditions are true.

$\angle 1 = 40°$
$\angle 2 = 50°$
$\angle 3 = 30°$
$\angle 4 = 60°$

$\angle FGJ = 30°$
$\angle FGI = 65°$
$\angle JGI = 35°$
$\angle IGH = 45°$

Therefore,
$\angle 1$ and $\angle 2$ are complementary $\angle$s.
$\angle 3$ and $\angle 4$ are complementary $\angle$s.

No two angles as drawn equal 90°. These angles as drawn are not complementary.

Construct pairs of angles that are examples or non-examples of complementary angles. Write the conditions for each. Exchange drawings with a partner and label the angles shown.

Note: Non-examples of geometric concepts embody some, but not all relevant characteristics and are useful in clarifying, "understanding" and identifying "misunderstanding." For example, orientation is an irrelevant characteristic, but is mistaken for relevant by many junior high school students.

Repeat the activities described above for supplementary angles.

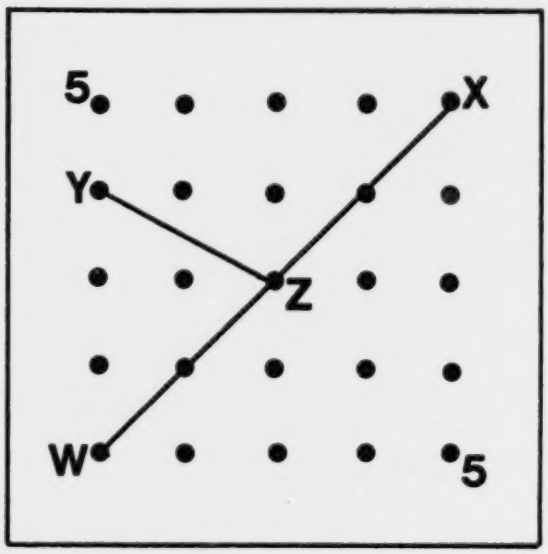
Complementary/Supplementary Angle Activity 2. THE SUPPLE COUPLE

PURPOSE: To construct and identify related angles using the media of geoboards to foster experimentation (Level 3)

MATERIALS: Geoboard, rubber bands and/or dot grid paper.

DIRECTIONS: 1. Do this activity as a directed lesson. Have students place a rubber band from (1, 1) to (5, 5), and another from (3, 3) to (1, 4).

Call (1, 1) W, (5, 5) X, and (3, 3) Z and (1, 4) Y.



Name the 2 angles using letters as shown. What kind of angles are they? Are they equal? What do the two angles form together?

2. Have students move the rubber band from Y to another point not on WX. Name the two angles. What kind of angles are they? Are they equal? What is their sum?

3. Discuss, then have students conclude that: If the sum of two angles is a straight angle or 180 degrees, then the angles are called <u>supplementary</u> angles.

4. Have students experiment with complementary angles by placing a rubber band from (1, 1) to (1, 5) and one from (1, 1) to (5, 1). Describe the angle formed. Place a third band from (1, 1) to any other point except (1, 5) or (5, 1). Describe the angles formed? What is their sum?

5. Discuss, then have students conclude that: If the sum of two angles is a right angle or 90 degrees, then the two angles are called <u>complementary</u> angles.

6. Have students experiment with other angle formations using the geoboard. Have students make true statements that accurately describe the relationships of pairs of angles formed.

Note: This activity applies the skill of naming points on a coordinate plane. See HSPT Skill #30 and related activities for how to name points on a coordinate plane.

Complementary/Supplementary Angle Activity 3. GEOMETRY THROUGH PAPER FOLDING

PURPOSE: To construct, analyze, and abstract properties using a simple media, waxed paper. (Level 3)

MATERIALS: Sheets of wax paper about 6" x 6" and a pencil

DIRECTIONS: While doing the following constructions, the edges or corners are not to be used as beginning places, but both sides of the wax paper may be used. When too many lines cause confusion use a clean sheet. Mark points and label them.

Construction a:

Label points X and Y on a sheet. Construct $\overleftrightarrow{XY}$ by folding. Find the midpoint of $\overline{XY}$ by folding. (Recall that $\overleftrightarrow{XY}$ means line XY, while $\overline{XY}$ means line segment XY).

Construction b:

Mark two more points, M and N, on the sheet. Construct $\overleftrightarrow{MN}$ and find the perpendicular bisector of $\overline{MN}$ by folding. (Perpendicular lines intersect at right angles).

Construction c:

Construct a line RS and mark a point P not on $\overline{RS}$. Fold a perpendicular through P to $\overline{RS}$.

Construction d:

Fold and mark a ray QT, $\overrightarrow{QT}$; then mark a point A not on $\overrightarrow{QT}$. Fold the ray QA. (Recall that the endpoint of a ray is named first when naming the ray). Bisect angle AQT or angle TQA, ($\angle$AQT or $\angle$TQA), by folding.

Construction e:

Construct an angle of 90°, right angle. By folding, construct an angle of 45° and an angle of 22 1/2°.

Identify the complementary angles constructed.

Identify the supplementary angles constructed.

a. label them: C = complementary, S = supplementary.

b. cut two complementary angles and form a figure with them. Be creative. How many ways can these figures be rearranged to form new figures?

c. cut two supplementary angles, form a figure. Follow directions for (b):

Use figures for discussion.

1. Are there an unlimited or a limited number of pairs of complementary angles in the figures formed? Why? Why not?

2. Must complementary angles share a common side? a common point? Why? Why not?

Complementary/Supplementary Angle Activity 4. CRITICAL THINKING

PURPOSE: To encourage development of mental imagery and abstract reasoning without concrete examples. This may be considered a transition from Level 3, Ordering-Abstraction, to Level 4, Deduction, and a prelude to Level 5, Rigor.

MATERIALS: None

DIRECTIONS: Show or say similar statements.

Example 1: Angle ABC is a right angle. Angle ABC is the sum of angle 1 plus angle 2.

What else do you know about angle ABC?*

If angle 1 measures 45°, what is the measure of angle 2?

How does angle 1 relate to angle 2?*

Example 2: Two angles $\angle$ PQR and $\angle$ JHK are supplementary. With this information, what else do you know about $\angle$ PQR and $\angle$ JHK? The measure of $\angle$ PQR is 72°. What is the measure of $\angle$ JHK?

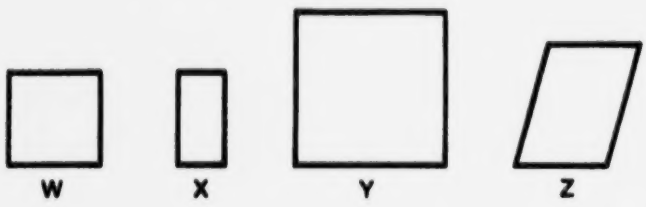
How does $\angle$ PQR relate to $\angle$ JHK?*

* Make a chart of students' responses for discussion and analysis in a follow-up lesson.

Skill #29

Identify similar figures. The condition for testing includes sketches of figures, two of which are similar.

Test item: What two figures appear to be similar?



A. X and Y
B. W and X
C. Y and Z
(D.) W and Y

HSPT, 1985

ENABLING SKILLS AND KNOWLEDGE

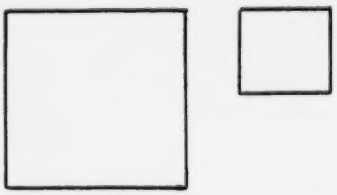
Give students the following information as needed:

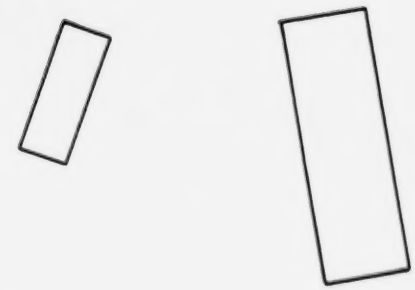
1. Angles of similar figures are equal even when the lengths of sides are unequal. Example: a large square and a small square each have four ninety degree angles. All squares are similar.

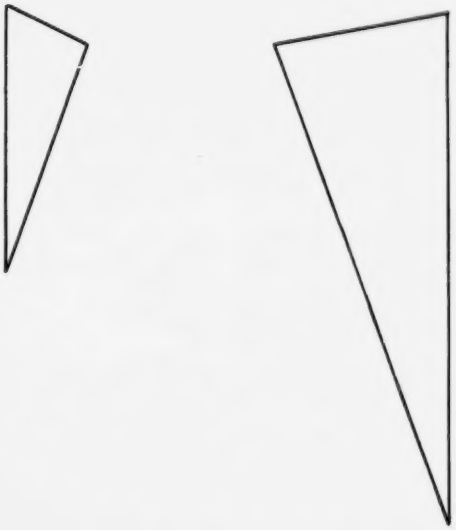
2. Position (orientation) of the figures does not change the size of the angles.

"Similarity of Triangles is a concept related to proportional reasoning, and it is a concept which many high school students find difficult to apply to mathematical problems." (Carpenter, et al., 1981)

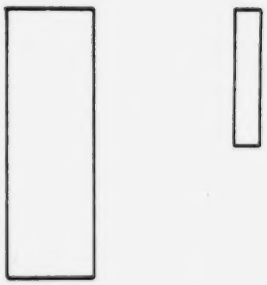
Examples of similar figures

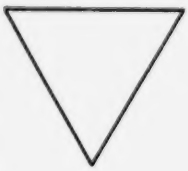






Non-examples of similar figures





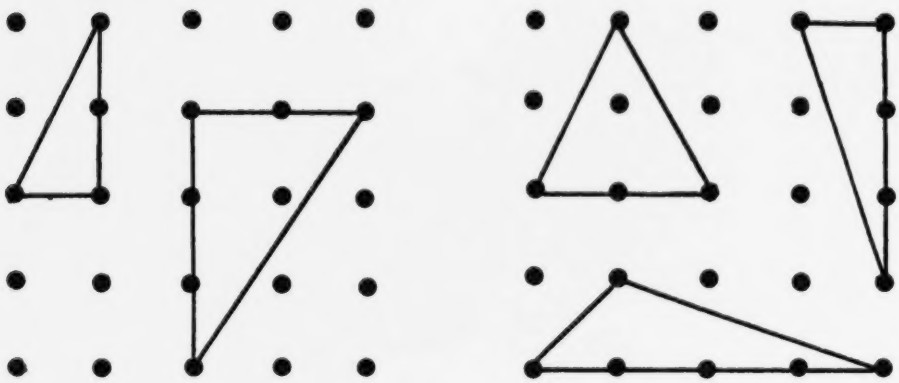
Similar Figures Activity 1. NO MATTER WHAT SHAPE YOUR TRIANGLE IS IN.

PURPOSE: To provide experimentation through the flexible media of geoboards used to create similar triangles and foster the development of formal reasoning.

MATERIALS: Geoboard, rubber bands, grid point paper.

DIRECTIONS: Form a triangle on the geoboard. Form another triangle which has exactly the same shape, but larger in size than the first triangle. Triangles that have the same shape, but not the same size are called similar triangles.

Shown below are diagrams of triangles. On the geoboard, copy these triangles and see how many others you can construct which are similar to the each triangle.



Perhaps you noticed that certain triangles which you constructed appeared to be exactly alike. Form a triangle around any 3 pins of the geoboard. Can you find 3 other pins which give exactly the same shape and size as the original triangle. Record your ideas on grid point paper. Triangles which have exactly the same shape and size are said to be congruent.

Similar Figures Activity 2. TANGRAMS

PURPOSE: To analyze and order properties of a figure by creating and manipulating whole/part components of figures, (Level 3, Ordering-Abstraction with transition to Level 4, Deduction).

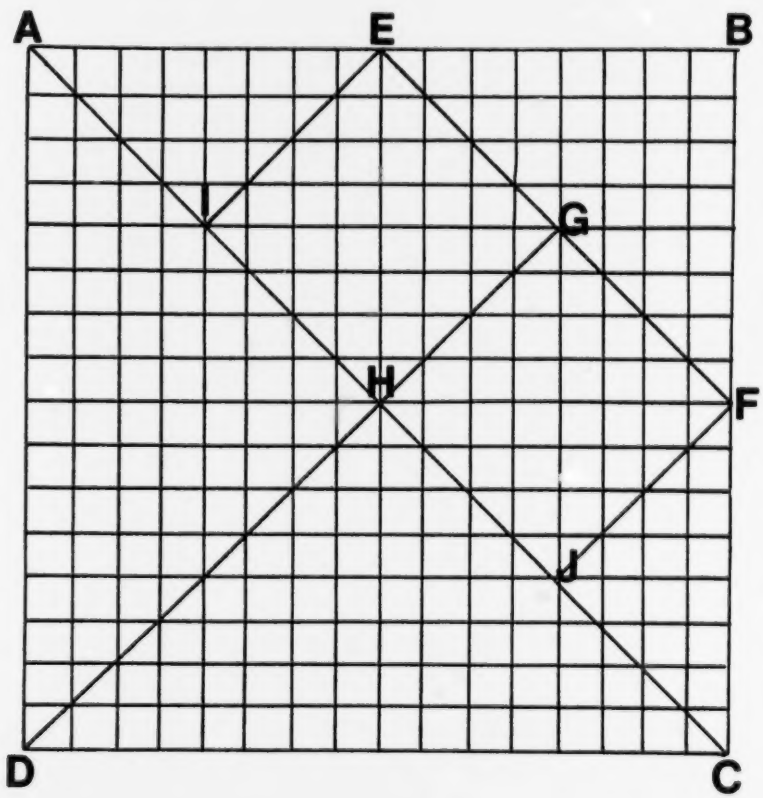
MATERIALS: Pattern and directions for 7 piece paper puzzle (TANGRAMS). Two sheets of grid paper of different sized squares (1 cm graph paper plus 1/4" graph paper). Transparency of centimeter graph paper.

DIRECTIONS: Demonstrate step-by-step using an overhead projector and transparent centimeter grid paper.

Follow directions as printed below. Use colored markers to connect points. Have students model each step.

Draw a square 16 units on each side. Label corners A, B, C, D as shown. Find mid-point of AB: Label it E. Find the mid-point of BC and label it F. Connect EF. Find the mid-point of EF, label it G. Draw line AC. Find the mid-point of AC label it H. Continue to find mid-points I and J, and connect as shown.

Make a chart and graph the number of triangles and names of each.



Before cutting TANGRAM shapes discuss:

A. Are any triangles similar? If so, name them.

B. Are there similar squares? If so, name them.

C. Are there similar rectangles? If so, name them.

D. Are there similar parallelograms? If so, name them.

Cut each piece on the drawn lines. Use shapes to trace patterns, pictures, and to form new shapes.

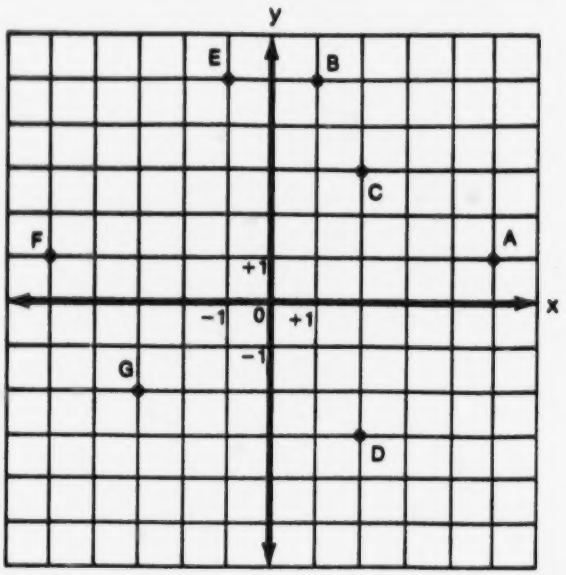
Optional:

Find the area of each piece by counting the squares. Find the area by measuring and using a formula. Make a second puzzle using different sized graph paper. (Suggested as homework.)

Skill #30

Identify points on a coordinate plane. The condition for testing is: given points on a coordinate plane.

Test Item:



Which point has coordinates of (1,5)?

A. A
(B.) B
C. E
D. F

What are the coordinates of point D?

A. (−2,3)
B. (2,3)
(C.) (2,−3)
D. (−3,2)

HSPT, 1985

Enabling Skills and Knowledge

Give students the following information as needed:

1. The intersection of each perpendicular line is a point which can be labeled with letters or numbers.

2. The name given to a point of intersection is "lattice point."

3. Points are read in pairs, the horizontal (left-right) named first, then the vertical (up-down) points.

4. The horizontal line is called the "X" axis, the vertical is "Y" axis.

5. Negative numbers are used to name points to the left of the vertical axis and below the X axis.

Coordinates Activity 1. PICTURING THE POINT

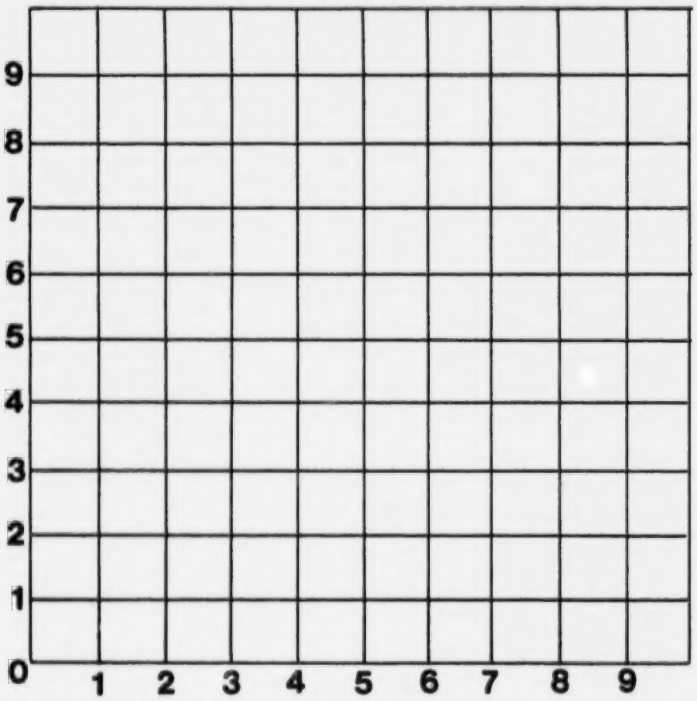
PURPOSE: This activity applies knowledge of points on a coordinate plane to practice recognition and analysis to form a picture. Students in van Hiele Level 1 or Level 2 benefit from this type of experience.

MATERIALS: graph paper, straight edge

DIRECTIONS: Locate the base line and a vertical line up the left side of the graph paper. Number the points on the base line 0 to 9 left-to-right (x axis). Number the vertical line 0 to 9 from bottom to top (y axis). Draw or trace a simple design: tree, star, etc. Locate the points where each of the lines of the picture touch the intersecting lines on the graph.

Name the points by counting the position 0 to 9 on the x axis first, then the position on the y axis.

Example: Points (3, 0) and (5, 0) are the base of the tree trunk. What are the coordinates of the tree top?



Variation: Increase the level of difficulty by showing both negative and positive points on x and y axis as in the HSPT test item. Give directions which combine negative and positive coordinates to form a picture.

Coordinates Activity 2. GRIDTOWN'S POINTS OF INTEREST

PURPOSE: To locate points following verbal/written directions. This activity requires analysis, ordering and abstraction .

MATERIALS: Coordinates Activity #2 Worksheet, Map of Gridtown, U.S.A. (See next page.)

DIRECTIONS: Demonstrate that if the streets in a city were all parallel and perpendicular, the corners could be identified just like points on a coordinate plane.

Locate the bank: Begin at Avenue A and 1st Street. Go north three blocks, east three blocks. Locate bank in the building of the S/E corner. What is the Avenue/Street location? (give coordinates)

Locate the library: Continue south two blocks. Locate the library in the N/W corner building. What is the Avenue/ Street location?

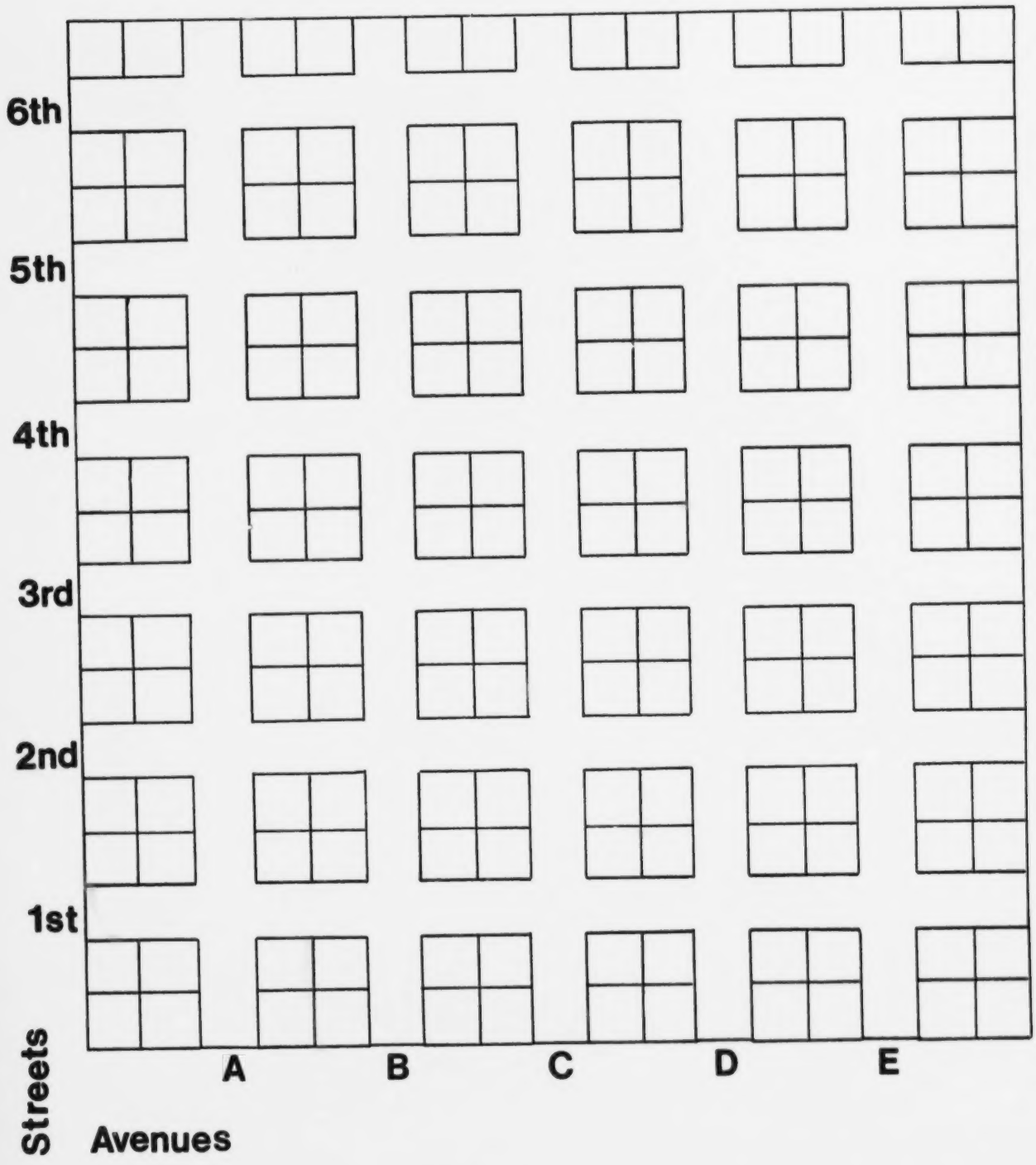
Locate the firehouse: It is on S/E corner of Avenue F and 1st Street.

Locate the police station exactly four blocks west. What is the address of the police station?

Find a friend's apartment at Avenue B and 4th Street. There is a flower in the window of her S/E apartment. Draw the flower on that corner.

Make up more directions and locate a theater, stores, doctor's offices, etc.

MAP OF GRIDTOWN, U.S.A.



Coordinates Activity 3. CLASSY* TRIANGLES FROM COORDINATES

PURPOSE: To construct triangles from an analysis and stated properties using flexible media. This experiment develops Level 3 thinking.

MATERIALS: Geoboard or five-point grid sheet, rubber bands.

DIRECTIONS: Use geoboards or a grid sheet to construct triangles with the following vertices. Each vertex is named as a point on a coordinate plane. After each set of vertices, write the type of triangle you have constructed.

1. (5, 2), (1, 3), (5, 4) _____

2. (1, 1), (2, 2), (5, 1) _____

3. (2, 2), (5, 2), (4, 5) _____

4. (1, 1), (3, 1), (2, 3) _____

5. (2, 3), (5, 1), (5, 4) _____

6. (2, 4), (4, 4), (3, 4) _____

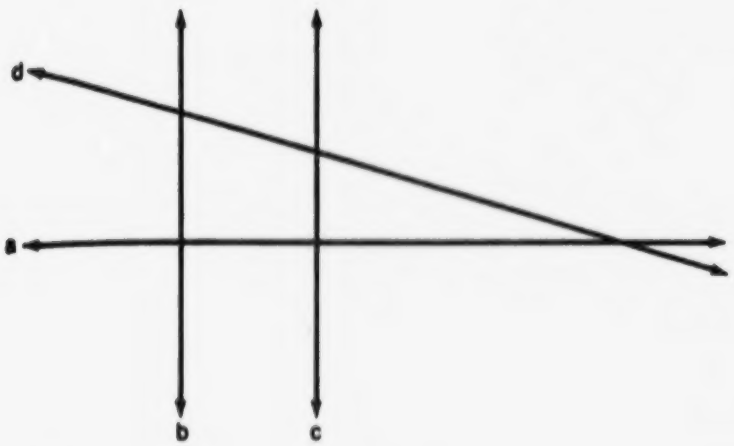
7. An equilateral triangle is a triangle with three equal sides. Try to construct it and other types of triangles on the 5 by 5 geoboard or graph paper. Name the coordinates for each triangle you construct.

* SUGGESTION: May be used to introduce triangle classification.

Skill #31

Demonstrate an understanding of parallel and perpendicular lines. The conditions for testing this skill include visual or verbal descriptions of two parallel lines cut by a perpendicular line.

Test Item:



Which two lines appear to be parallel?

(A.) b and c
B. a and d
C. a and b
D. c and d

Which two lines appear to be perpendicular?

A. c and d
B. a and d
C. b and c
(D.) a and c

*HSPT, 1985

Enabling Skills and Knowledge

Give students the following information as needed:

1. Lines are composed of points.

2. Lines extend in either direction from a point.

3. Two straight lines that are drawn equally distant will remain equally distant however far they are produced.

Line Activity 1. CONSTRUCT A LINE

PURPOSE: To deduce from experimenting that a line is a series of points; a Level 2 developmental activity.

MATERIALS: Paper or transparency and pen.

DIRECTIONS: Model steps 1, 2 3.

1. Put a dot on the paper.
2. Put another dot on the paper, about an inch or two centimeters distant from the first point.
3. Locate the point midway between the other two points and place a dot at this point.
4. Continue locating the mid-point between each pair of dots until no space remains between dots.

Line Activity 2. CONTINUING PAIRS OF LINES

PURPOSE: To experiment by drawing continuing lines leading to classification of straight lines as parallel or intersecting. (Level 3 to Level 4, Deduction.)

MATERIALS: Toothpicks, straight edge, blank transparency for teacher.

DIRECTIONS:

1. Drop two toothpicks on the overhead as the starting line segments. Or,

   Draw two non-parallel line segments (less than 5 cm) anywhere on the sheet.

2. Extend each line straight, in both directions to the edge of the paper or until they cross.

3. Picture the lines continuing in the same straight direction into "space" until they cross.

4. Predict the point of "crossing" (intersection) based on the relative positions of the starting line segments.

5. Discuss: What determines the point of intersection?

Line Activity 3. PLUMB LINES AND PERPENDICULARS

PURPOSE: To understand the property of a perpendicular line, a Level 3 activity, using experimentation with plumb lines.

MATERIALS: Plumb line, paper to fold, (T square, optional).

DIRECTIONS: Drop a plumb line (string or thread with a sinker attached will work) Figure 1 . This "line" intersects the line on the horizontal plane at right angles. Use a T square, or make a right angle model: Fold an 8" x 11" or larger sheet of paper (randomly, irregularly, not parallel or perpendicular to any sides) Figure 2. Then fold over carefully matching up the line of fold Figure 3. This "model" or T square can be used to "measure" the right angle formed as the plumb line intersects with the line. The "pendant" formed by the plumb line is the root word of perpendicular.

Example:

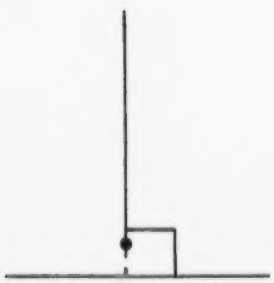


Figure 1

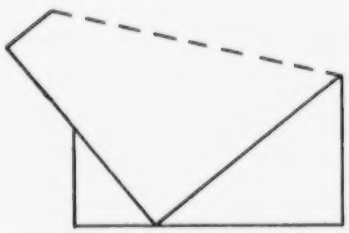


Figure 2

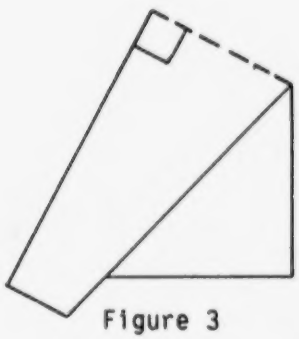


Figure 3

Line Activity 4. WORD - SYMBOL - FIGURE MATCH

PURPOSE: To provide practice using vocabulary and language skills to communicate properties of concepts, Level 3.

Memory Clues:

A. Parallel has two "ls" side by side which are "lines" that are equidistant. Think of them as representing parallel lines.

B. Perpendicular is from "perpendiculum" which means "plumb line." Pendent comes from the same root and means "to hang."

Symbols are: $\perp$ perpendicular

$\perp_s$ perpendiculars

$\parallel$ parallel

$\parallel_s$ parallels

MATERIALS: 3x5 index cards, tape

DIRECTIONS: Have each student:

1. Make a set of word cards and a set of symbol cards. Assign pairs to work together.

2. One student from a pair tapes the word on an example of parallel or perpendicular lines in the classroom. Examples: Place the word card for perpendicular at the cross bars of a window frame.

3. When all word cards are placed, give a signal for the partner to find the card and add the correct symbol card. Teacher checks the match. The activity can be repeated using symbols first. It takes only about five minutes for all to participate. Variations are encouraged.

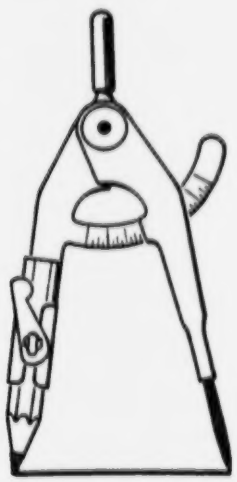
Line Activity 5. STRAIGHTEDGE AND COMPASS CONSTRUCTION

PURPOSE: To provide experience with measurement tools to actually construct perpendicular and parallel lines.

MATRERIALS: straightedge, compass.

DIRECTIONS: Demonstrate, then have students:

1. Draw a line (I) with a straightedge. Mark point (O) on the line.
2. Place the point of the compass on point O. Mark off the same distance on both sides of point O by intersecting the line (I) with arc points A and B.
3. Open the compass a little. Place the compass point on A. Make a mark above point O.
4. Keep the same opening, place the point of the compass on point B and make a mark above point O. The marks (arcs) intersect at point C.
5. Draw line CO. Line CO is perpendicular to line AB at point O.
6. Draw another perpendicular line on line I to construct parallel lines.



SECTION TWO

COMPUTING TASKS

The following skills' #21, #23, #24, #26, #27, and #28 are grouped in section two because they require computation. Students see the parts of the figures and come to understand the relationship of these parts by observing, actual measuring, and drawing the figures or making models of them. The skills are those in the van Heile Level 2 development described in the Introduction section, A Framework For Presenting Instruction. The students may see the relationships of the parts of the figures presented, but not fully grasp their meaning. Therefore, experiences that require deductive reasoning need to be provided. Even with evidence of previous instruction and learning, step by step instruction, with attention to enabling skills, is necessary for most of the students in this age group.

Further Discussion of Skill #23

In order to master an understanding of complementary and supplementary angles, students need to acquire not only the labeling skills described previously, but also the ability to see and express the numerical relationship of one angle to another and the relationship of the angles to a "whole" of 90° or to the "whole" of 180°. This part of the task requires computing sums, or subtracting a measure of one angle from the "whole." For example, when the measure of a given angle is 70° and the student is asked to determine the measure of the complementary angle, the student computes the difference from 90°. The knowledge of the appropriate vocabulary, the ability to see the relationship of a given angle to the whole, the selection of the operation, and the ability to accurately subtract seventy from ninety are the steps required to solve this type of problem. If students have difficulty, it is necessary to know at what step the difficulty begins. Having students verbalize their thinking is one way of assessing difficulty.

The next five skills are presented together because they require computation and analysis as described in van Heiles' developmental Level 2. Students who are alert to the parts of a figure, and understand that the figures act as bearers of their properties, are said to be in the van Heile developmental Level 2. Instructional strategies must be selected appropriate to students' developmental levels, as is indicated by Driscoll in reporting Kidder's study.

> "Any geometric problem that requires a student ... to hold more than one variable in mind at a time will severely tax the child who is in the preformal reasoning stage. To illustrate, Kidder conducted a study that required 13-year-olds to consider the invariants when a triangle in a plane is rotated through a fixed

angle. The students' answers were a mixture of correct and incorrect conclusions, and Kidder ascribed this to their having to focus on one variable (e.g., length of sides) and then another (e.g., the positions of the triangle vertices with respect to the origin of the rotation). In particular, they generally showed an inability to conserve length in the task--that is, to understand that in the rotation of a triangle the side lengths do not change." (Driscoll, 1983)

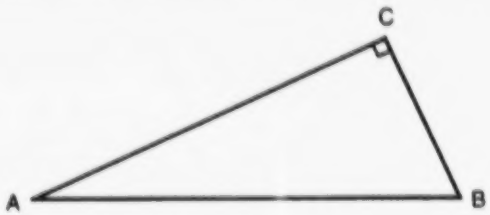
Skill #24.

Understand that the sum of the measure of the angles of a triangle is 180°. The test condition is either visually or verbally given the measure of two of the angles.

Test items:

A triangle has two 35° angles. What is the size of the third angle?

A. 30°
B. 35°
(C.) 110°
D. 290°



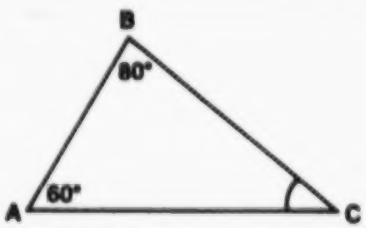
If the measure of angle B is 65° and angle C is a right angle, what is the measure of angle A?

A. 15°
(B.) 25°
C 115°
D 155°

HSPT, 1984

A triangle has a right angle and a 30° angle. What is the size of the third angle?

A. 120°
B. 90°
(C.) 60°
D. 30°



What is the measure of angle C in the above triangle?

A. 20°
(B.) 40°
C. 60°
D. 140°

A triangle has two 40° angles. What is the size of the third angle?

A. 10°
B. 80°
(C.) 100°
D. 140°

HSPT, 1985

Enabling Skills and Knowledge

Give the students the following information as needed.

1. A triangle is a triangle in any orientation, with any combination of length of sides and size of angles.

2. Obtuse, acute or right angles are formed at the three vertices.

3. Angles can be measured using a protractor and the measure is stated in degrees.

4. Students are able to measure angles with a protractor (students are given opportunity to practice this skill as needed).

Instructional Implications

The incorrect responses (distractors) chosen by students who took the HSPT in 1984 indicate that students chose answers that were the sum, the difference or equal to the measures of the angles given.

On one item, when the measures of two angles were given, one third of the students selected the sum of the measure of the two angles.

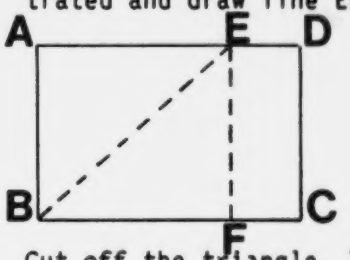
Instruction must reinforce the concept that the sum of the three angles is constant, regardless of the given measure of any one or two angles in the triangle.

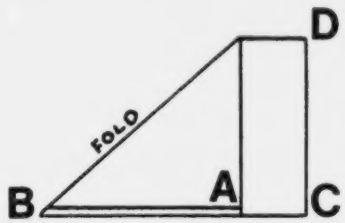
Triangle Activity 1. ADDING ANGLES

PURPOSE: To engage students in a strategy of experimentation, Level 3, by manipulating angles cut from a triangle.

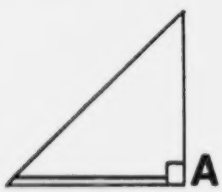
MATERIALS: Large paper rectangle, scissors.

DIRECTIONS: Label the corners of the rectangle A, B, C, D. Fold corner A so that edge AB lines up with edge BC. Make a mark along edge BA of the triangle. Label points E and F as illustrated and draw line EF.



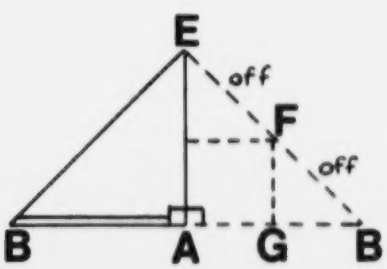


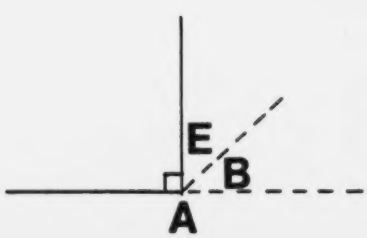
Cut off the triangle. The vertex A is a right angle. Angle A has not changed measure. It has changed position. Place the triangle with angle A on point F.



Flip the triangle over on line EF. Angle BFC is a straight angle that measures 90° + 90° = 180°. Be sure that the three angles of ABE are marked.

Fold point B to point A and point E to point A. Cut off each small triangle, as shown. Place the cut off triangles along the extended base (BFC) at point F so that the vertex B and vertex E form a new right angle. Angles A, E, and B show that $\angle A + \angle B + \angle C = 180°$, as illustrated.





Triangle Activity 2. STRAIGHTENING OUT A TRIANGLE

PURPOSE: To promote deduction, Level 4, through manipulation of components of triangles.

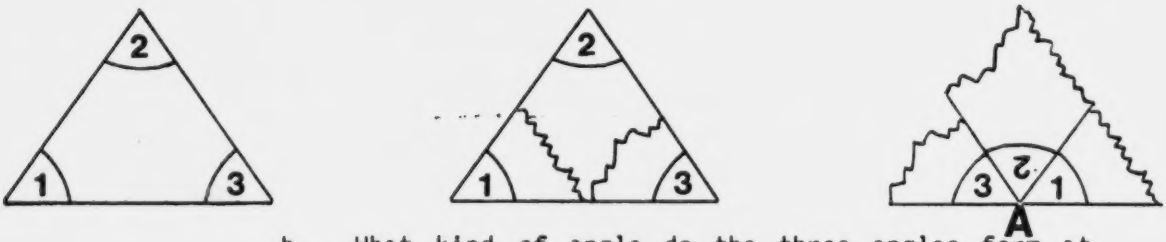
MATERIALS: Scissors, paper and ruler

DIRECTIONS:

1. Draw 3 different kinds of triangles (isosceles, equilateral, scalene). Number the angles of each triangle 1, 2, 3, respectively.

2. For each triangle:

   a. Cut or tear off the 2 bottom corners of one of the triangles and put the angles side by side as shown below.



   b. What kind of angle do the three angles form at point A?

   c. What is the sum of the angles?

3. What can you say about the sum of the angles of any triangle?

Source: Adapted from Middle School Mathematics Laboratory, West Virginia Department of Education, 1975.

Triangle Activity 3. CLEVER CLUES

PURPOSE: To reinforce vocabulary and descriptions of the relationships of parts of geometric figures.

MATERIALS: Crossword puzzle sheet

DIRECTIONS: Complete puzzle

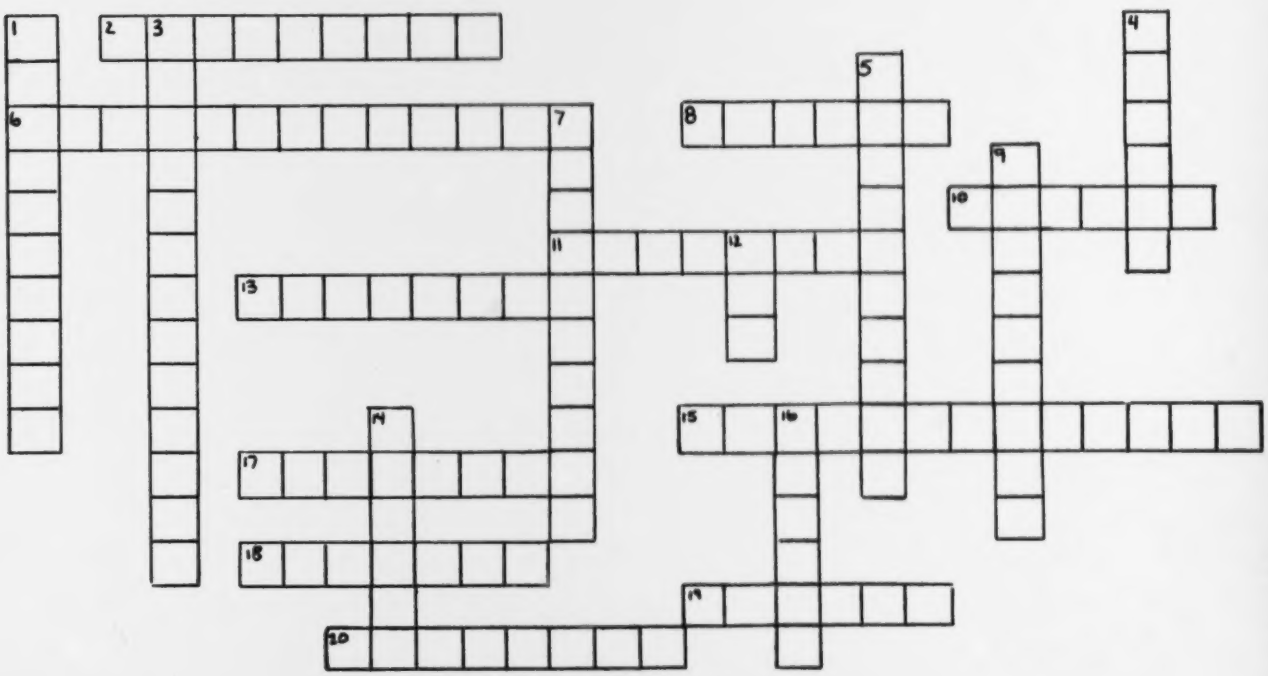
ACROSS

2. A triangle with 2 congruent sides
6. 2 lines that meet at a right angle are _____.
8. To divide in half.
10. A polygon that has all points connected by a line inside the polygon is said to be _____.
11. A line divided by a point is a _____.
13. A dot exactly in the center.
15. Two angles whose sum is 90 degrees are _____.
17. The union of 2 half planes with a common edge to form a ____ angle.
18. A statement proved true is a _____.
19. An angle more than 90 degrees.
20. Two angles with a common ray.

DOWN

1. The side opposite the right angle in a right triangle.
3. Two angles whose sum is 180 degrees.
4. Measure an angle in ____.
5. An angle less than 90°.
7. An angle of 90°.
9. Same size and shape.
12. Not a hypotenuse, but a ____.
14. Part of a minute.
16. Part of a degree.

Answers on next page.



CLEVER CLUES ANSWERS TO CROSSWORD PUZZLE OF GEOMETRY TERMS

Across

2. isosceles
6. perpendicular
8. bisect
10. convex
11. half line
13. midpoint
15. complementary
17. external
18. theorem
20. adjacent

Down

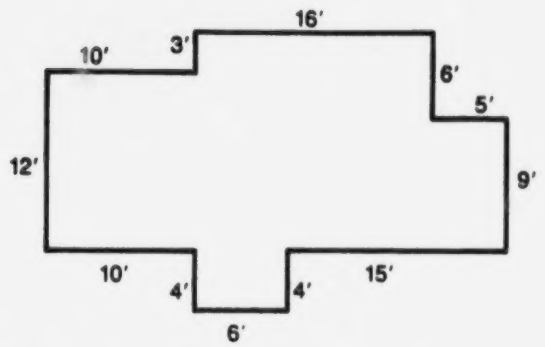
1. hypotenuse
3. supplementary
4. degree
5. acute angle
7. right angle
9. congruent
12. leg
14. second
16. minute

Source: Adapted from Middle School Mathematics Laboratory, West Virginia Department of Education, 1975.

Skill #26

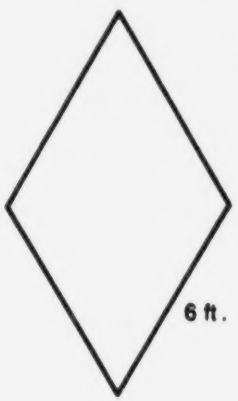
Find the perimeter of a polygon. Testing conditions are given a regular polygon and the length of one side; or, given an irregular figure and the length of all sides.

Test items:



A. 94 feet
B. 96 feet
(C.) 100 feet
D. 104 feet

What is the perimeter of this rhombus?
(All sides are of equal length.)



A. 12 feet
(B.) 24 feet
C. 36 feet
D. 72 feet

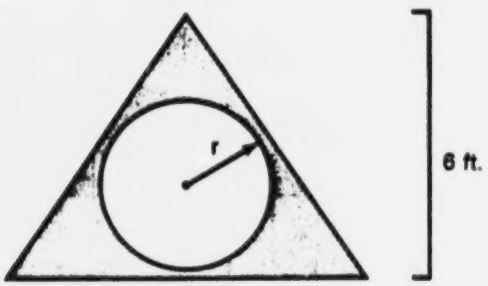
HSPT 1985

Skill #27

Compute the area of a specified region. The test conditions are give two figures (rectangle, triangle or circle and a shaded region) with no formula.

Test item:

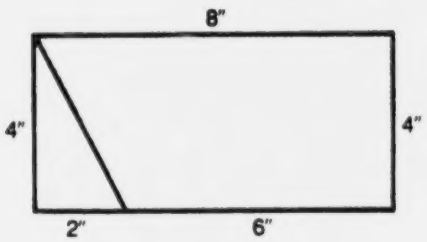
What is the **area** of the shaded portion of this figure?



$r = 2, \quad \pi = 3.14$

(A.) 11.44 sq. ft.
B. 17.72 sq. ft.
C. 25.44 sq. ft.
D. 36.54 sq. ft.

What is the **area** of the shaded portion of this rectangle?



A. 12 square inches
B. 24 square inches
(C.) 28 square inches
D. 48 square inches

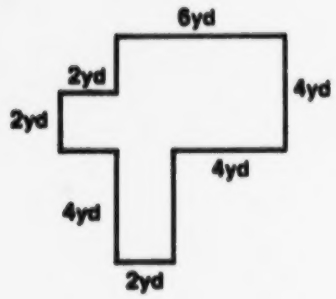
HSPT, 1985

Skill #28

Compute the area of an irregular figure. The test conditions are with no formula given and using a combination of rectangles.

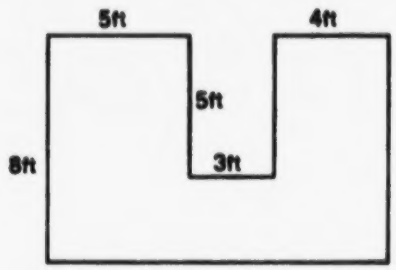
Test Item:

What is the area of this figure? (All angles are right angles.)



A. 40 square yards
(B.) 36 square yards
C. 32 square yards
D. 28 square yards

What is the area of this figure? (All angles are right angles.)



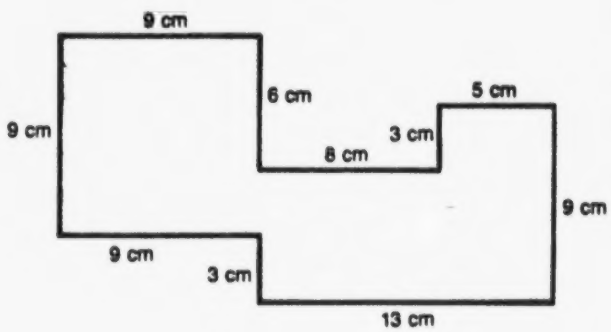
A. 96 square feet
B. 87 square feet
(C.) 81 square feet
D. 50 square feet

HSPT, 1984

What is the area of this figure? (All angles are right angles.)

A. 40 square inches
B. 56 square inches
(C.) 60 square inches
D. 80 square inches

What is the area of this figure? (All angles are right angles.)



A. 74 square centimeters
B. 148 square centimeters
C. 150 square centimeters
(D.) 174 square centimeters

HSPT, 1985

Enabling Skills and Knowledge

Give students the following information as needed:

1. Plane figures have both perimeter and area.

2. Perimeter is the length around a closed curve and is measured in linear units.

3. Area is the amount of surface inside a figure and is measured in square units, or the number of squares.

4. Arithmetic operations can be carried out on unit measures. (Students can perform such computations)

Since confusion of the skills of area and perimeter appears to account for many errors on district, statewide and national tests, these skills will be considered together. Instructional activities that develop understanding of the differences of these concepts will be suggested.

> "If conservation of length and conservation of volume are weak points for individuals who have not reached the level of full formal thought, it would seem likely that conservation of area would be a weak point, too. Indeed it is, as Szetela learned. He gave seventhand eighth-graders the task of deciding whether or not deforming the perimeter of a shape affects its area. Of course, it usually does (for example, changing a square into a rhombus with the same side-length does not change the perimeter, but it changes the area), but Szetela found that the pre-formal thinkers in this group of subjects were inclined to believe that areas are invariant under such transformations of perimeter." (Driscoll, 1983)

Instructional Implications

The incorrect responses (distractors) indicate that as many or more students computed the perimeter instead of the area than those who chose the correct response to test items requesting total area.

Instruction must discriminate between the concept of area and the concept of perimeter, and the rationale for the selection of the appropriate operation. Instructional focus on the correct use of units, i.e. square units for area measure, would prevent many common errors.

Perimeter/Area Activity 1. STRINGS AND THINGS

PURPOSE: To experience length measured by a line. To relate length to perimeter of a figure or shape.

MATERIALS: String, objects

DIRECTIONS: Use an arm span length of string. Demonstrate that it can be used to

outline the outer edge of a flat object (book, picture, paper plate, etc.)

wrap around edges of the faces of a three-dimensional object (box, vase, globe, etc.)

The distance around is called the perimeter. It is the measure of a line around an object or figure. Tell students that perimeter goes around the rim.

Have students use a string to outline a plane figure or wrap around objects and describe the perimeter in terms of the length of the string.

An example of a discription might be, "The perimeter of this figure is about half the length of the string."

Perimeter/Area Activity 2. EQUAL PARTS AS UNITS OF LINEAR MEASURE

PURPOSE: To experience use of a non-standard unit of measure, and develop sub-units as equal parts of a unit.

MATERIALS: String, colored pens, objects to measure

DIRECTIONS: Fold the string into a number of equal segments: three, four, five, six, etc.

Mark the string at these folds to identify equal segments.

Make up a name for the units.

Measure each object. Record the measure to the closest unit names.

Optional: Use a metric (cm) or standard units (inch) calibrated tape measure after students have demonstrated some knowledge and understanding of non-standard units.

Perimeter/Area Activity 3. GEOBOARDS AND PERIMETERS

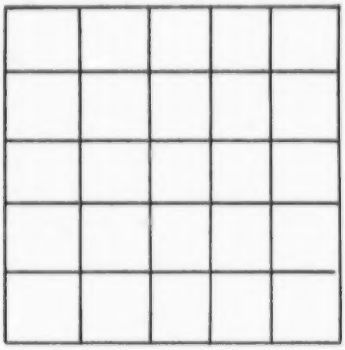
PURPOSE: To experience perimeter and area using the geoboard.

MATERIALS: Geoboards, rubber bands, grid paper that matches geoboard in number and position of points, and transparent geoboards.

DIRECTIONS: Place the rubber band on the points of the geoboard to form: triangle, square, rectangle, octagon and other polygons which have been introduced. The perimeter is the path of the rubber band.

Record the perimeter of each figure on grid paper.

Estimate, then count the number of linear units.



EXTENSION ACTIVITY: Have students estimate the square units of surface area of each shape recorded.

Perimeter/Area Activity 4. THE SHAPE EXCHANGE

PURPOSE: To experiment with a given area by constructing various figures. This activity promotes development from Level 3 to Level 4.

MATERIALS: Point grid, graph paper or geoboard.

DIRECTIONS: Select one or two from the following list of tasks, and assign each to pairs of students. Some tasks are simple, some complex. Assign simple tasks to the less confident students. After students complete tasks, pairs may exchange their "products." Then they may tell the criteria used, how it was formed, what it is, and state conclusions.

A representative of the pair presents the construction and the oral description to the class. Tasks may also be done in teams or individually.

1. Construct a square, 4 units on a side.

2. Construct a square, 4 square units.

3. Construct a triangle, 4 square units.

4. Construct a rectangle, 3 square units.

5. Construct a rectangle, 2 units on one side, 3 units on the other side.

   a. What is the area of this rectangle?

   b. What is the perimeter of this rectangle?

   c. Divide the rectangle into two parts. Are the areas of the two parts equal? If they are not equal, which is larger? How can you express this (one area $>$ other area; one area $<$ other area; one area = other area)? Say it. Write it.

   d. Divide the rectangle into thirds vertically by means of one colored rubber band. Divide the area into halves horizontally using a different colored band. Can you find 1/6 of the total area? 2/3? 5/6? Can you show on the geo-board that $1/2 \times 1/3 = 1/6$?

6. Repeat number 5 now by constructing a rectangle whose area equals 12 square units. What other mathematical relationships can you discover? Can you show on the geo-board that $5/6 - 1/3 = 1/2$? $1/2 - 1/3 = 1/6$?

7. Construct a triangle of area equal to 3 square units.

8. Construct a parallelogram of area equal to 3 square units.

9. Construct a trapezoid of area equal to 5 square units.

10. Construct a pentagon. What is its area?

11. Construct a hexagon of area equal to 4 square units. Can you construct a square inside the octagon? What is its area?

12. Construct an octagon. What is its area? Can you construct a square inside the octagon? What is its area?

13. Construct triangles of area equal to 1 square unit, 2 square units, 3, 4, 5, 6, 7, 8 square units. Are any of these impossible?

14. Make up others.

Perimeter/Area Activity 5. CONNECTING COMPUTATION TO CONSTRUCTIONS

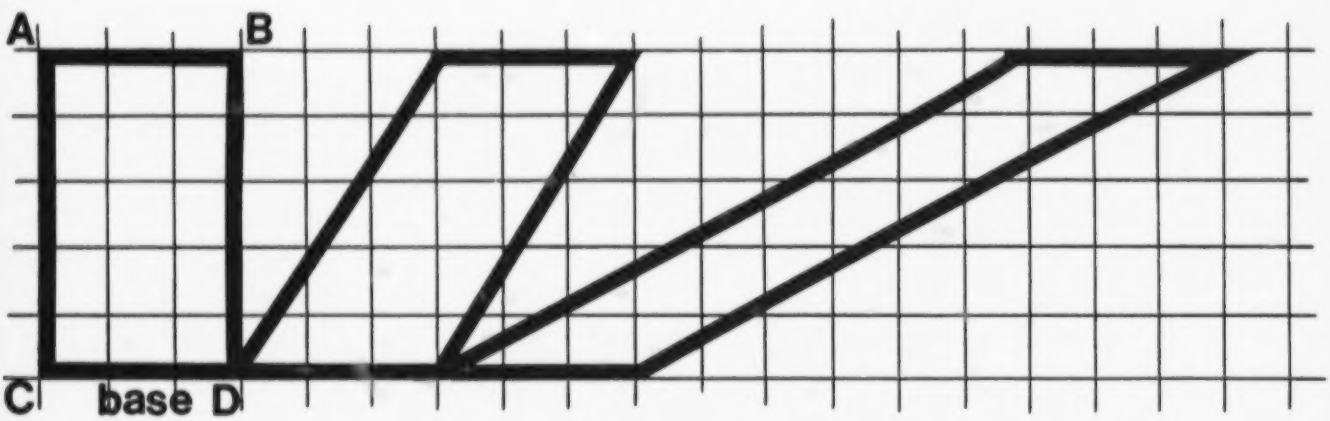
PURPOSE: To provide experiences to develop formal reasoning and the role of the formula for finding area.

MATERIALS: Transparent centimeter grid, similar grid (graph paper) for each student.

DIRECTIONS: In the upper left corner draw a rectangle whose base is about two to four units and height is four to eight units. Have students follow teachers' modeled drawings step-by-step. Tell students to write the number of square units in the enclosed region. Ask, "How did you get it?" Accept only a complete statement. Have a student record all complete statements. Keep asking, "Is there another way?" until no one volunteers. Then, "How do the statements compare? Are they all correct?

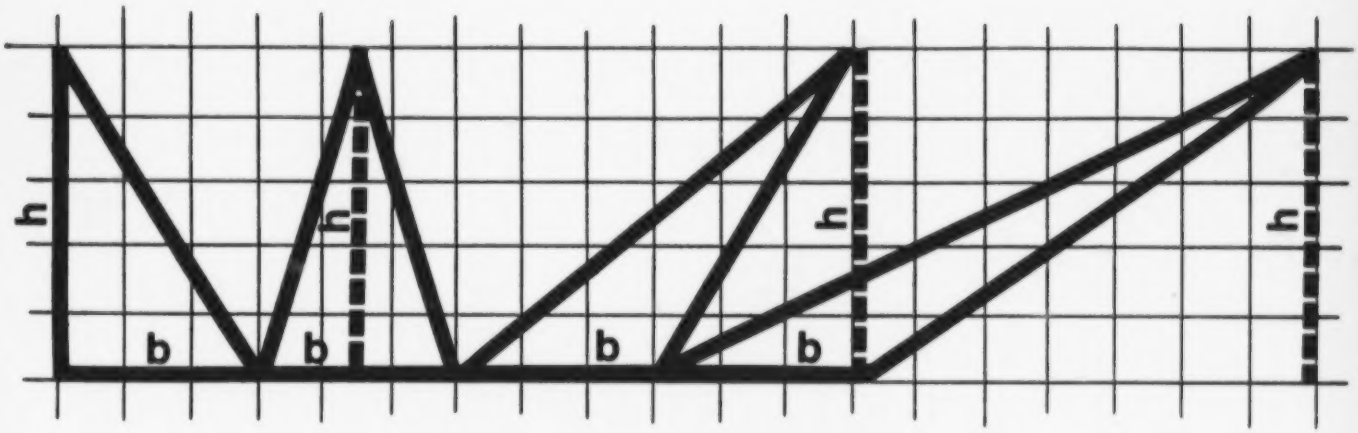
Teacher: "Area is the number of square units in the region." Count the number of squares in each row." Or, "Count the number in one row, and multiply the number of rows."

Which of the students' statements most closely describes one of the formulas, A = L x W, area equals length times width, or A = b x h, area equals base times height? Label base and height on the rectangle. Continue to locate a point x number of units to the right on the top of the figure as illustrated. Connect so that the base line of each new figure is continuous. Draw a dotted line perpendicular to the bases and mark the right angle of each figure. The new figures are parallelograms as illustrated.



Area of a parallelogram = base x height in square units.
Count the number of square units in each figure shown.
Compute the area of each figure using the formula.
Compare the answers. State conclusions.

Repeat this procedure for a triangle as illustrated.



Area = 1/2 base x height

Compare area of rectangle to area of triangle $a = \underline{\text{base x height}}$
$a = \underline{\text{base x height}}$, respectively.
2

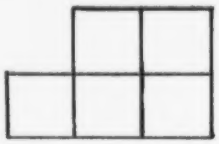
Perimeter/Area Activity 6. CONSTANT AREA, CHANGING PERIMETER

PURPOSE: To provide an experiment with area measure by manipulating squares to promote Level 4 thinking, deduction.

MATERIALS: five congruent squares, grid paper, ruler (optional)

DIRECTIONS: Place the five squares to form a figure such that at least one whole side of each square touches the side of another square.

For example:



is ok.

is not ok.

Trace the figure on graph paper.
Label the figure #1.
Record the area: _____ sq. _____
Record the perimeter: _____

Rearrange the squares to form a new figure.
Trace the figure on graph paper. Label it #2.
Record the area: ___ sq._____
Record the perimeter: _____

Continue to arrange, trace, label and record area and perimeter of each new formation. There are twelve arrangements.

Do not tell students how many. Let them work in pairs or teams. A possible group project could be to cut out and mount all twelve arrangements ordered from shortest to longest perimeter.

This activity requires time to think, compare, and sequence conclusions. It is appropriate as a long term assignment, a project for groups or teams, or as an independent homework assignment. It may be necessary to give this hint to your students. If two figures can be matched up after they are cut out, they are the same arrangement.

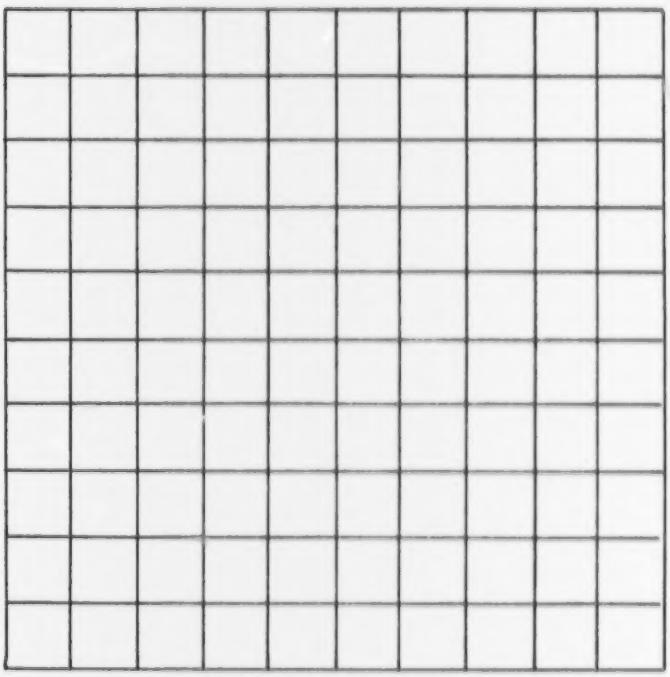
Perimeter/Area Activity 7. CONSTANT PERIMETER, CHANGING AREA

PURPOSE: To provide concrete experiences with constant perimeter.

MATERIALS: A string (not stretchy) about 20 cm, transparent grid sheet, grid sheet for each student, compass.

DIRECTIONS: Measure the string. Knot the ends to form a flexible closed curve. Place the string curve on a grid sheet. Form the largest rectangle possible. Describe it.
Count the squares in the region. ____ sq. ___
Record largest square area: ____ sq. ___
Form a rectangle: Count square units. Record.
Form a circle: Count square units. Record.

Have students state conclusions.
What is the relation of the area to the perimeter?
Which figures have the least area?
Which figures have the most area?



Skill #21.

The student will be able to add, subtract, multiply, and divide measures using metric and U.S. standard units. The test conditions include examples of weight, liquid and linear measures; denominate numbers; regrouping within the system; problem solving format; converting within the system-including square units.

Test items:

Rachel ran 10,000 meters in the morning and 17,500 meters in the evening. If she ran this distance each day for 5 days, how many kilometers did she run?

A. 13.75 kilometers
*B. 137.5 kilometers
C. 1,375 kilometers
D. 137,500 kilometers

HSPT, 1985

A truck driver delivered a shipment of 9 cartons of school supplies. Each carton weighed 13 pounds, 11 ounces. How much did the shipment weigh altogether?

A. 125 lbs. 3 oz.
*B. 123 lbs. 3 oz.
C. 117 lbs. 11 oz.

HSPT, 1984

---

27 oz. + 36 oz. + 62 oz. =

*A. 7 lbs. 13 oz.
B. 10 lbs. 5 oz.
C. 12 lbs. 5 oz.
D. 125 lbs.

HSPT, 1985

Enabling Skills and Knowledge

Students must know or be given:

1. Computational algorithms for whole numbers, decimals and fractions.

2. The system of converting within the metric system and the standard U.S. units.

3. The names and the corresponding measures for length, volume, and mass in both metric and standard U.S. units.

4. The correct symbols (metric), or abbreviations (U.S. standard) for each measurement unit.

5. The correct tools of measurement: scales, rulers, calipers, measuring cups, etc.

6. Metric prefixes:

			Base Units			
			meter			
kilo	hecto	deka	liter	deci	centi	milli
1000	100	10	gram	.1	.01	.001

Metric measure prefixes in common use are underlined.

Within the organization of this monograph, this skill falls in the category of Type 2 tasks, computing units. As test items, the skills required are similar to those for solving word problems. Each example given may include the objectives for several lessons. The hands-on opportunities to measure things using the units for length, volume, and mass in both systems is the only strategy which will promote understanding of measurement. In this way students will become facile in estimating and performing appropriate measurements. The benefit of these strategies has carry-over for life skills, as well as higher-order thinking skills.

> "Measurement activities using real tools and objects, making comparisons, using symbols for measurement terms, using objects to demonstrate fractional representations, are all ways to introduce understanding of proportion." (Phillips, 1978)

Instructional Implications

The incorrect response (distractor) most frequently chosen by students in 1984 indicates that students selected an appropriate operation but applied the operation to only one of the two given units. For example, most students: multiplied yards, but not feet; multiplied pounds, but not ounces; subtracted meters from meters, but did not subtract centimeters converted from meters.

Instruction must focus on the conversion of subunits within each measurement system. Practice problems that are presented in the context of applications should be encouraged.

Measurement Activity 1. CHOOSING UNITS

PURPOSE: To develop the ability of students who have had limited experience in measuring so they learn to select the appropriate metric or standard units.

Linear units are selected for their approximation to the size of the objects to be measured. Examples:
millimeters are used to measure the diameter of a pearl;
centimeters are used to measure the length of a belt;
meters are used to measure the dimensions of a room;
kilometers are used for distances, depth, and evaluation.

MATERIALS: Objects and pictures of common objects which would be measured with specific units. Example: picture of a road with a caption noting the distance in kilometers.

DIRECTIONS: List the units for length, volume and mass for both metric and standard units. Have students collect objects appropriate for each unit. Write situational "problems" for each unit of measure that require computation. Increase the level of difficulty as students become comfortable with appropriate units.

Note: When adding inches with a sum greater than 12, "conversion" to feet and inches is common practice. When adding centimeters to a sum greater than one hundred, the sum is correctly written 109 cm or 1.09 meters. 1 meter 9 centimeters is not common, and is not recommended by Systeme International (SI).

Measurement Activity 2. THE ONE-METER DASH

PURPOSE: To add centimeter units in a game format, regrouping them to "decimeters" to build a meter.
Variation: roll dice for inches to a yard.

MATERIALS: Two metersticks, cuisenaire rods, two dice or number cubes, pencil and paper, if notation is desired.

Each team should have ten orange (that is, ten 10-rods) and a supply of each of the shorter rods. Color paper strips can be substituted for Cuisenaire rods.

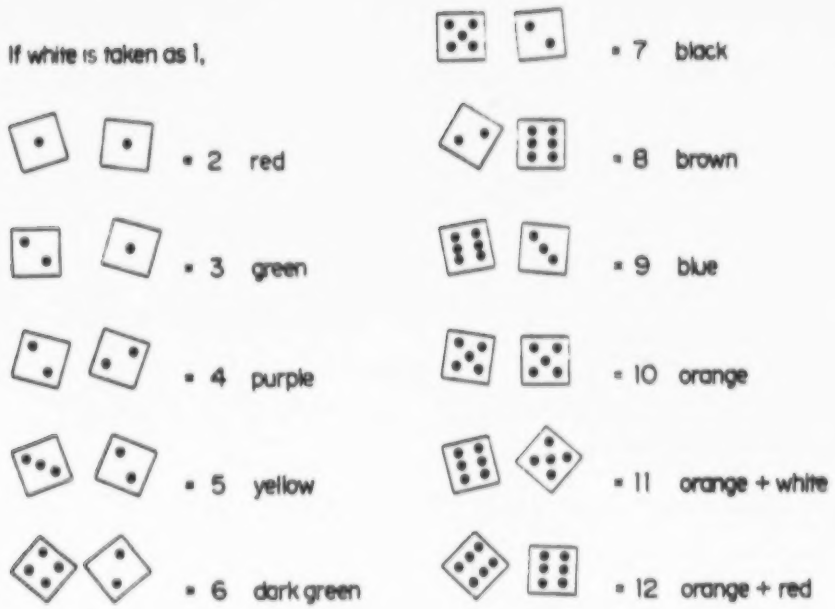
PREPARATION: The game can be played with two players or with two teams of up to three players on each team.

- o The two teams face each other across a table.
- o Two metersticks are placed on the table between the two teams. The 0-cm point of each meterstick is at the 100-cm point of the other. Placed together in this manner, each stick may be read as a number line by one side.
- o A supply of Cuisenaire rods.
- o A "staircase" of ten Cuisenaire rods, which forms a "memory bank" for the players.

DIRECTIONS: The teams take turns and the play rotates among the players on the teams.

1. A player rolls the dice and translates the number of spots that turn up to a Cuisenaire rod. (See illustration) Beginning at 0 cm, these rods are placed in a train along the team's meterstick.
2. At any time this growing train must contain only one non-orange rod. Whenever a train of two non-orange rods appears, it must be traded for either.
   a. one rod whose length matches the two rods, or
   b. an equivalent two-rod "train" consisting of one orange rod plus one other rod.
3. All orange rods must be first in the train starting at 0; only one non-orange rod may follow the train of orange rods.
4. The first player or side completing "the one-meter dash," (forming a train of ten orange rods) according to the rules wins the game.

Illustration



Here are several variations of the game. You may think of more.

"The One-Meter Retreat," where each side begins with the meter-stick filled with ten orange rods and subtracts the rod(s) indicated by rolls of the dice. The first side to have no rods remaining on the meter-stick wins.

"The Two-Meter Dash," where a train of twenty orange rods must be completed first to win. The numbers on the dice or number cubes may be multiplied in this form of the game.

"The Two-Meter Retreat," where each side begins with twenty orange rods and subtracts the appropriate rod(s). Again, the numbers on the dice may be multiplied. First side to subtract all the rods wins.

Source: The Arithmetic Teacher submitted by Mattie J. Brooks, an eighth grade mathematics teacher in Chicago, Illinois.

Measurement Activity 3. VISUALIZING CAPACITY

PURPOSE: To experiment with liquid units to develop reference points for visualizing liquid capacity.

MATERIALS: Plastic two-liter soda bottles, milk cartons (gallon, half-gallon, quart, pint, and "school milk" half-pints) paper cups, other unbreakable containers.

DIRECTIONS: Have students show the following by pouring and then recording.

a) What is the relationship of a liter to a quart? (Answer: A little more than a quart.)

b) How many of each type of small measuring container is required to fill each next larger measuring container? Make a chart for U.S. standard units and one for metric units.

c) A liter is equal to 1 000 millilliters. What is half a liter equal to? What unit in the U.S. standard measure is similar to the half-liter? (Answer: pint)

d) How many millilliters is one-fourth liter? What unit in the U.S. standard measure is similar to the quarter-liter? (Answer: one cup)

About how much coffee does a styrofoam cup hold? Estimate in millilliters. Estimate in ounces.

Recommended: Use colored liquid.

Measurement Activity 4. ESTIMATING AND COMPUTING WITH MEASUREMENT UNITS

PURPOSE: To provide experience estimating.

The following two student worksheets suggest estimating and computing activities for the classroom. Estimation using measure- ment units is the goal in each of these activities.

Students cannot increase their skill in estimating measurement units without concrete experiences.

Such experiences are worth the trouble if you consider the truth of the Chinese proverb which says, "I do, and I understand!" Use the suggested format "estimate then measure" and make up collections for length and liquid, as well as mass.

MATERIALS: The activity worksheets may be reproduced for students to use.

GUESS AND WEIGH

MATERIALS: Scales, gram weights, a collection of common items (ten items are suggested).

DIRECTIONS: Look carefully at each item.
Feel the mass of the 500 gram weight or the 100 gram weight. (1000 grams is called a kilogram)
Think: Is the object less than or more than 500 grams?
Estimate the weight in grams.
Write your estimation in the first column of your paper.
Measure the object using the scale.
Write the exact weight in grams and/or kilograms.

	Estimation	Actual Weight
1. twenty-five pennies		
2. one golf ball		
3. one tennis ball		
4. one cake of soap		
5. fifteen large paperclips		
6. bag of beans		
7. bag of marbles		
8. one hundred white rods		
9. package of Jello		
10. box of margarine		

*** "Supersleuths" only *** Write the difference between your estimate and the actual weight of the object. Add the total grams for each column.

SIZING UP THE SITUATION

Circle the best answer:

1. If an elevator can hold 10 average eighth graders, is its weight limit

   8 kg, 80 kg, 800 kg?

2. You could probably eat

   10 g, 500 g, 1000 g of pancakes and drink;
   10 ml. 500 ml, 1000 ml of juice for breakfast.

3. If you filled your empty car tank with gas, you would put in

   8 liters, 80 liters 800 liters.

4. You probably weigh about

   4 kg. 40 kg, 400 kg.

5. An individual-sized carton of milk holds about

   2 ml, 25 ml, 250 ml.

6. Your books for three classes together weigh approximately

   3 kg. 30 kg, 300 kg.

Try this experiment:

Find and record the weight (mass) when a one liter container is filled with: water, juice (or some other liquid), rice, sand, and dried beans. Be sure to weigh the container when it is empty and subtract its weight from the total.

	water,	juice,	rice,	sand,	beans
weighs	___g;	___g;	___g;	___g;	__g;

7. Based on the results of the experiment, does one milliliter of water weigh

   1g, 10g, 100g?

   Discuss your answers to the questions with your classmates. Be ready to prove that your answers are correct.

Source: "Ideas," The Arithmetic Teacher, January, 1981.

SECTION THREE

APPLYING A FORMULA TASKS

These skills, #25, and #27 are grouped together because the solutions to these problems require the memorization or the application of a formula, a third type of task required of students. On the HSPT, the problem of finding the area of a specified region is complicated by the overlapping of one figure on another, or having embedded figures. Solving such a problem requires memorization of formula, association with the relationships of the two figures, and awareness of the relationships of the parts of each figure. The origin of the elements of the formula are not apparent from the information given.

The tasks required for this type of problem can be accomplished by students who have reached Level 4 described in van Hiele. Deductive methods develop through experimentation. While the ability to memorize formulas can be prompted by instructional strategies, understanding the role of theorems and proofs is still not a fully developed stage for most middle school students. Therefore, activities to allow experimentation, as well as those that promote memorization, must have a place in the repertoire of teaching strategies.

The relationship of radius to circumference of a circle is one of the activities that builds a basis for estimating the measure of circumference and area. The formulas which include π increase the level of abstraction for students. Providing easy reference to written formulas and then providing a need for using them in many contexts will motivate students to memorize the formulas.

For the HSPT test items, students are expected to know the formulas for the area of a circle, area of a rectangle, and area of a triangle. These need to be memorized. On the HSPT, the formula are given for the area and/or volume of a cone, pyramid and sphere. The type of gridboard activity described in Perimeter/Area Activity #5 is designed to promote understanding of the development of the formula for computing area.

The development of geometric thinking from Level 1, Recognition, through Level 3, Ordering and Abstraction, prepares the student to reason formally, to understand and use formulas. The following skills and related developmental activities correspond to these levels of development. They also describe informal geometry, which, unlike formal geometry, does not require development at Level 5, Rigor.

Skill #25.

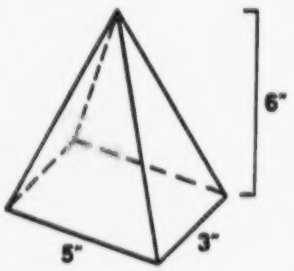
Find volume of cone, pyramid, rectangular solid or sphere. The conditions for testing this skill are with a figure presented, definitions of variables given, (pi equals 3.14), and with the formulas:

Test Items:

Find the volume of the pyramid with a rectangular base using the formula

$V = \frac{B\,h}{3}$

(B = area of the base, h = height)

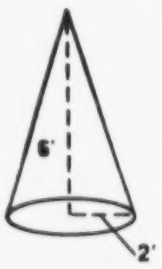


(A.) 30 cubic inches
B. 32 cubic inches
C. 90 cubic inches
D. 96 cubic inches

Find the volume of this cone using the formula

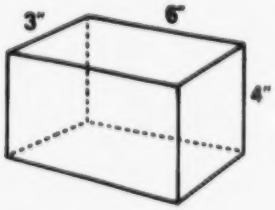
$V = \frac{\pi\, r^2\, h}{3}$

(Let $\pi = 3.14$, r = radius of the base, and h = height.)



A. 12.56 cubic feet
(B.) 25.12 cubic feet
C. 50.24 cubic feet
D. 75.36 cubic feet

Find the volume of the rectangular solid below.



A. 13 cubic inches
B. 26 cubic inches
C. 60 cubic inches
(D.) 72 cubic inches

HSPT, 1984

Enabling Skills and Knowledge

Give students the following information as needed:

1. Volume describes the amount of space in the interior of a solid.
2. A cube must have six square faces.
3. The unit for measuring volume is a cube whose edges are a linear unit.
4. The measure of volume is expressed as the number of cubes that will fill the interior of the solid.

Instructional Implications

The incorrect responses (distractors) chosen most frequently for problems when a formula was given were answers for incomplete substitution and operations using the formula. When the formula wasn't given (for a rectangular solid) students added the dimensions given.

Instruction must focus on the concept of volume as well as the procedural and computational steps for the given problem, with or without a formula. Frequent opportunities to estimate and measure volume will provide students with a point of reference and help them avoid unreasonable answer choices.

Volume Activity 1. CONCEPT OF INTERIOR

PURPOSE: To provide experimentation with volume measure to support the development of abstraction and transition to deduction.

MATERIALS: Unit cubes, sugar cubes (be sure they are cubes), a collection of dice (sets of the same size), centimeter or inch cubes, (Rubiks or Crazy Cubes may be used), small empty boxes, a liter box or a solid cubic decimeter (10cm x 10cm x 10cm).

DIRECTIONS: This activity may be demonstrated first by the teacher, then students working in pairs should carry out this activity and its variations until estimations are close to the actual count.

- o Identify the cube as the unit of measure.
- o Compare the unit to the interior of an empty box.
- o Estimate the number of cubes that would completely fill the box.
- o Fill the box as completely as possible.
- o Estimate the number of partial cubes over or under capacity.
- o Remove the cubes and count them.
- o Write the number of cubes and estimate the incomplete cube units.
- o Rearrange that number of whole cubes to form a different shaped rectangular solid.
- o Measure the length of the edges of the empty container and of the newly formed arrangement using the edge of the cube as one unit of length.

The empty box measures: _____ length _____ width _____ height

The new rectangular solid measures: _____ length _____ width _____ height

Multiply the measures for each object. Example: height x length x width

Write the computation for each activity. Write the formula for volume of a rectangular solid. Substitute the measures from this activity for the terms of the formula.

Volume Activity 2. CUBIC CONCEPT DEVELOPMENT

PURPOSE: To promote ability to visualize three-dimensional shapes from two-dimensional diagrams. This is an abstract level of thinking which promotes deductive reasoning as in Level 4.

MATERIALS: Sets of 30 cubes each, worksheet.

DIRECTIONS: Have students work in pairs or teams using the following worksheet to:

1. Construct solids using cubes by looking at a two-dimensional picture of a solid (the way it is shown in texts and tests).

2. Follow written directions to construct three-dimensional figures.

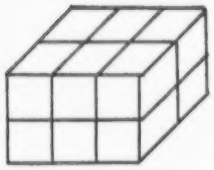
3. Use oral and written language to communicate and record mathematical experiences.

Teacher: Demonstrate each of the experiences listed above. Assign teams. Prepare worksheet. (A sample is provided).

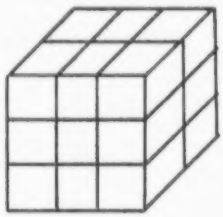
WORKSHEET FOR CUBIC CONCEPT DEVELOPMENT

1. Get a box of cubes

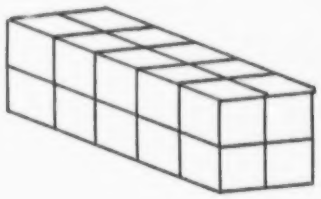
2. Use the cubes to construct the following solids. Record the number of cubes needed for each one.



a.



b.



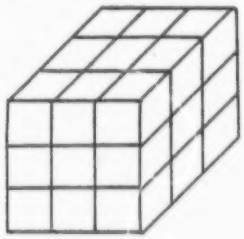
c.

a. _____

b. _____

c. _____

d. _____



d.

3. Make a solid box using 18 cubes. Record the length _____

the width _____

the height _____

4. Make a solid box using 30 cubes. Record the length _____

the width _____

the height _____

5. Make a solid box using 24 cubes. Record the length _____

the width _____

the height _____

A solid box can be built with 13 cubes. Build it.

Think of other problems like this one for which there is only box possible. Write them down.

Adapted from: Action Math Associates, Inc. 1974

Volume Activity 3. PART-WHOLE CONCEPT

PURPOSE: To provide experience using language, visualizing a construction, manipulating "volume" units and computing. This is an activity that promotes transition from Level 3, abstraction, to Level 4, deduction.

MATERIALS: Cubes, student worksheet.

DIRECTIONS: Demonstrate using eight small cubes to form a larger cube that the length of an edge is two "cube edges." Record and compute volume.

Assign students to tasks. (Sample of a worksheet is provided.)

STUDENT WORKSHEET FOR VOLUME PART-WHOLE CONCEPT

1. Get a box of cubes

2. Use eight small cubes to construct a larger cube. How long is each edge? Record. Draw Compute Volume

3. Use 27 small cubes to construct a larger cube. How long is each edge? Record. Draw Compute Volume

4. Construct a larger cube than before. How many cubes did you use?
   Record. Draw Compute Volume

5. Do you have enough cubes to construct the next largest cube? How many do you need?
   Record Draw Compute Volume

6. Make a chart of your constructions. Is there a pattern?

Number of Cubes	Length of Edge	Volume

7. <u>Do not build the following cubes.</u> Estimate the number of small cubes needed to build a cube with an:

edge of 11	edge of 20	edge of 35	edge of 100
_____ cubes	_____ cubes	_____ cubes	____ cubes

Adapted from: Action Math Associates, Inc. 1974

Volume Activity 4. ANATOMY OF A FORMULA, WHAT MAKES IT WORK?

PURPOSE: To use deductive reasoning, Level 4, to understand the role of formulas. This activity for volume of a cylinder, cone or sphere, given the formulas and the value of pi.

MATERIALS: A paper cylinder (orange juice can), ruler, paper cup or cone, and sheets of graph paper

DIRECTIONS: Present the formula for the volume of a right circular cylinder.

Write the measure of each term given in the formula.
Example: $V = \pi r^2 h$. Translate into words.

The volume (V) of a right circular cylinder is equal (=) to the product of the area of the base, a circle, (πr^2) times the altitude of the cylinder. [Multiply area of base times the measure of altitude (h) in the formula.]

Use graph paper to trace the circle of the base.
Cut, fold in half to indicate the diameter.
Fold in half again to show the radius.
The area of a circle is an approximate number of square units.

Count the linear units approximating the radius and multiply the terms πr^2.
Compare the number of square units on the drawn circle.
Measure the height of the cylinder using the same units on the graph paper as a linear measure.
Multiply the terms.

Have students "visualize" the volume expressed as the number of cubes inside the cylinder.

Have students fill the cylinder with liquid.
Pour this into a calibrated beaker.
Read the result.
Compare computation to the result in the beaker.

- Present the formula for volume of a cone. Translate the formula into words and into a model.

Cut a circle and form a cone or cut apart a cone. Lay it flat.

Measure the surface area using the formula and compare it to a similar shape on graph paper.

Estimate area, estimate volume based on experiences.

Compare visible physical properties to the terms of the formulas. Try estimation problems such as:

> The volume of an ice cream cone with a base radius of 5cm and an altitude of 8cm given the formula $V=\frac{\pi r^2 h}{3}$
>
> without computing. Set up a range within which the answer will fall. Estimate the value of π as 3+. (a little more than 3)

- Present the formula for finding the volume of a sphere and procede to develop the activities as described for cylinder and cone. Try estimation problems such as:

  The volume of a sphere with a radius of 3mm (a bead) given the formula $V=\frac{4}{3}\pi r^3$: Eliminate unreasonable answers.

SECTION FOUR

SIMPLIFYING TASKS

Three of the skills, #25, #27, and #28, on the HSPT which have been discussed are also examples of irregular figures or require a complex sequence of operations. One problem-solving strategy that can be applied to the instruction of these skills is to transform a complex problem into a simpler one by breaking it down into manageable parts. This strategy can be applied to problems posed by these three HSPT skills.

To compute the area of an irregular shape, skill #28, it is appropriate to find the area of the simple shapes which make up the irregular shape. Students must be guided to "see" the irregular shape as a composite of simple shapes. Then, they can more easily compute the total area.

To compute the area of a specified region, skill #27, it is necessary to identify that region relative to the region surrounding it, or taken from it, especially if one figure is superimposed or embedded, as in the test item.

To find the volume of three-dimensional figures, skill #25, the process is more easily understood when the figures are broken into two-dimensional figures. The relationship of all of the parts of the figure can then be considered, and the relationship of area to volume is demonstrated.

These strategies have applications in other situations as well. They need to be practiced as strategies to solve problems such as are tested on the HSPT, but also as strategies that students will apply to other mathematical and non-mathematical situations.

CONCLUSION

Junior high school students have the potential for learning these informal geometry and measurement skills. Adequate instructional time can be allocated to the study of measurement and geometry during the junior high school curriculum. For students at the middle school, the study of geometry serves as a connecting bridge between concepts and computation learned in elementary school and the opportunity for applications in measurement and geometry. Students whose learning style is dependent upon visual stimulus often discover that they can "see" mathematical relationships for the first time when they study geometry. A good foundation in geometry has been shown to have a positive effect on later learning of formal geometry, algebra and calculus. Zalman Usiskin has identified three reasons these geometry skills should be included in the curriculum prior to formal mathematics at the secondary level.

i) Geometry connects mathematics and the real world.

ii) Geometry enables ideas from other areas of mathematics to be pictured. For example, geometry lends visual aid to subjects like advanced algebra and calculus, and hence makes them more accessible to learners.

iii) Geometry is an example of a mathematical system--in fact, one of the earliest examples available to students. (Usiskin, 1980)

Lastly, because the ideas, activities and resources presented here are meant to prompt curriculum planners and teachers to further planning in the area of measurement and geometry, a matrix showing informal geometry topics, environment (materials) and instructional resources is included in this monograph in Appendix A. This matrix is an adaptation of a more complete one which includes such topics as tessellations, polyhedra, etc. The resources recommended here provide more extensive activities than is reasonable in this monograph. You are encouraged to make use of these resources to further improve the teaching and learning of measurement and geometry.

In Appendix B, the skills in the Measurement/Geometry Cluster are listed in numerical order from #21 to #31, with the state cluster mean scores for 1984 and 1985. The average percent of correct responses for each skill that was tested each of the years is also given. Teachers and curriculum leaders may find this helpful for instructional planning.

Suggestion for local use: Compare your scores on each skill with the state averages and with your district scores on sequential years.

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APPENDIX A

APPENDIX A

Informal Geometry Resources: Topics x Environments

Numbers identify references which follow.

ENVIRONMENT

TOPICS	GRAPH PAPER	GEOBOARDS/ DOT PAPER	BLOCKS CUTOUTS SHAPES	DRAWINGS/ CONSTRUCTIONS	PAPER FOLDING	LOGO
VISUALIZATION	7 12	12	12 18 31 36	4 12 31	14 26	2
PROPERTIES OF 2D SHAPES	3 12 29	3 9 12 15 29 20	3 10 12 16 17 27 28 32 33 36 39 40 42	8 12 17 23 37 41	26	1 2 22 24 25 34 43
GEOMETRICAL DESIGN	5 6 12 39	5 12 38	27 30 35 39	4 5 8 11 13 18 21 30 37 38 41	12 14 26	1 2 22 24 25 34 43
MEASUREMENT	12	9 12 15	12 28			24

INFORMAL GEOMETRY TOPIC/ENVIRONMENT RESOURCES

*Reference Numbers correspond to numbers on the Topic x Environment chart.

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APPENDIX B

APPENDIX B

HSPT MEASUREMENT AND GEOMETRY CLUSTER SKILLS
Skill identification numbers refer to HSPT Skill Array-Mathematics

State Cluster Mean 84 - 85 50% 64%	State Average % Correct 1984	State Average % Correct 1985	Local District 84 85
21. Add, subtract, multiply and divide measurements	41.6	39.3	
22. Identify and define acute, obtuse and right angles	*N.T.	80.5	
23. Identify complementary and supplementary angles	42.3	43.0	
24. Understand sum of angles in triangle	50.3	60.5	
25. Find volume of cone, pyramid, rectangular solid or sphere	58	45.0	
26. Find perimeter of a polygon	N.T.	78.5	
27. Compute area of a region	N.T.	29.0	
28. Compute area of irregular figure	33.0	29.0	
29. Identify similar figures	N.T.	90.0	
30. Identify points on coordinate plane	N.T.	50.5	
31. Understand parallel and perpendicular lines	N.T.	58.5	

*Not Tested

Curriculum Questions:

- Which skills present the most difficulty for your students?
- Where are these skills introduced and reinforced in the curriculum before grade nine?
- Are these skills taught and tested the way they are tested on the HSPT?
- Which skills will receive the focus of your instruction?

FEEDBACK SHEET

Improving Mathematical Skills Assessed on the High School Proficiency Test

MEASUREMENT AND GEOMETRY

The Department of Education would appreciate your reaction to this document. Please answer the questions below and add any other comments after you have used this document for more than four weeks. Cite examples where appropriate. You may photocopy this form for your response.

Teaching Assignment or Title _____ Grade Level _____

Number of Years Teaching _____ Number of Years Primarily Teaching Math _____

Certification(s) _____

A. How did you obtain this resource book?

Workshop _____
Central Office _____
Principal _____
Other Teacher _____
Other (specify) _____

B. What training (assistance) did you receive in the use of this document? Would you like to have additional training?

C. Please describe any specific changes you have made in teaching measurement and/or geometry as a result of using this resource book.

D. What specific changes, if any, would you recommend for this resource book? Cite specific activities.

E. With whom have you shared this resource book?

Teacher(s) specify grade level _____
Principal/supervisor _____
Department chairperson _____
Other(s) _____

F. Other observations and remarks:

(Optional)

Name: _____

Address: _____

_____

Please return this feedback sheet to:

Bureau of Cognitive and Academic Skills Development
New Jersey State Department of Education
CN 500
225 West State Street
Trenton, New Jersey 08625